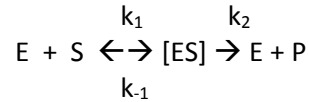


Derivation of the Michaelis-Menton Equation

1. Assume a reaction equation in which enzyme and substrate bind reversibly and then go on to form product. Assume an initial reaction, so there is very little product, making the second back reaction negligible.



2. The initial rate (v_0) of the reaction is

$$v_0 = k_2[ES]$$

3. The problem is that we cannot know $[ES]$ because it forms during the reaction. But we can make the steady state assumption that it is a low concentration that doesn't change because the rate of its formation is equal to the rate of its breakdown.

$$k_1[E][S] = k_{-1}[ES] + k_2[ES]$$

4. But we also don't know $[E]$, which is the concentration of free enzyme, because some of the enzyme is bound. We know that the total enzyme concentration is the sum of the free and bound enzyme.

$$[E_T] = [E] + [ES]$$

5. Substitute total enzyme in to the steady state and rearrange to solve for $[ES]$

$$k_1 (E_T - ES)[S] = k_{-1}[ES] + k_2[ES]$$

$$k_1 [E_T] [S] = (k_{-1} + k_2) [ES]$$

$$[ES] = \frac{k_1[E_T][S]}{k_{-1} + k_2 + k_1[S]}$$

6. Rearrange this expression to isolate the rate constants, then replace those constants with a new constant called the Michaelis constant, K_M .

$$[ES] = \frac{[E_T][S]}{\frac{k_{-1} + k_2}{k_1} + [S]}$$

$$[ES] = \frac{[E_T][S]}{K_M + [S]}$$

7. Substitute the expression of $[ES]$ from point 6. Into the rate equation from point 2. k_2 can also be called the catalytic constant, k_{cat} , because it represents the rate constant for reaction.

$$v_0 = \frac{k_2[E_T][S]}{K_M + [S]} = \frac{k_{cat}[E_T][S]}{K_M + [S]}$$

8. The maximum velocity, V_{max} , is equal to the total enzyme concentration times the catalytic constant. Substitute V_{max} into the equation to get the final form of the Michaelis-Menton equation.

$$v_0 = \frac{V_{max}[S]}{K_M + [S]}$$