1. Section 2: Nuclear Energetics

The energy stored in atomic nuclei is more than a million times greater than that from chemical reactions and is a driving force in the evolution of our Universe. The energy radiated by our Sun is the consequence of nuclear reactions that occur in its core. One of the greatest hopes for clean, abundant energy in the future is in the nuclear fusion reactor, which utilizes similar reactions to produce electrical energy. The elements that make up our terrestial environment are the products of nuclear reactions generated during the phases of Stellar Evolution. One of these elements, ^{235}U, is the fuel for nuclear power reactors. And miniature power sources used in many remote-sensing devices operate with the energy provided by radioactive decay.

The goal of the present section is to define the energetic terms necessary to discuss nuclear phenomena, especially the factors that govern nuclear stability. Not all combinations of protons and neutrons are able to form a unique nucleus, just as all combinations of atoms do not necessarily form stable compounds; e.g. two He atoms do not form He_2. A central theme of this chapter is to provide the basis for answering the question: why do some neutron/proton combinations exist and others do not? The answer lies in nuclear energetics.

<u>The Basics</u>

Nuclear energetic calculations are simplified relative to chemical thermodynamics in that for most applications, the entropy is zero (stellar interiors being a notable exception). For the purposes of this course, the basic thermodynamic equation is

$$E = Mc^2 \qquad \text{(with thanks to A. Einstein).} \qquad \text{(Eq. 2-1)}$$

As indicated in the introductory section, mass M is expressed in units of **atomic mass units u (or amu)** and energy E in terms of MeV. In this context, the value of c^2 is

$$\mathbf{c^2 = 931.494\ MeV/u.}$$

Make sure you always have access to this conversion factor as it will reoccur frequently.

Exercise: using the definition of u in grams and MeV in ergs, show that
1u = 931.494 MeV.

Thanks to Einstein, when discussing nuclear energetics, mass and energy can be treated interchangeably, either in units of u or MeV/ c^2 , as illustrated in Table 2-1.

Table 2-1

Symbol	Description	Mass in grams	Mass in u	Mass in MeV/ c^2
M_p	mass of proton	1.6726×10^{-24}	1.00728	938.272
M_e	mass of electron	9.1×10^{-28}	5.484×10^{-4}	0.511006
M_H	mass of 1H atom	1.6735×10^{-24}	1.00783	938.783
Mn	mass of neutron	1.6748×10^{-24}	1.00865	939.550

Two features of Table 2-1 should be noted. First, the masses of the proton and neutron are ~1830-1840 times larger than the mass of the electron, indicating the mass of the atom is concentrated in its nucleus. And second the mass of the neutron is greater than that of the proton, which has important implications for our Universe.

In order to perform nuclear energetic calculations, it is more convenient to work in units of MeV, rather than mass. In order to simplify calculations, we define a quantity called the **mass defect Δ** (sometimes called the mass excess):

$$\Delta = (M - A)\, c^2 \qquad \text{(units are in MeV)} \qquad \text{(Eq. 2-2a)}$$

or the mass can be calculated from

$$M = A + \Delta / c^2 \qquad \text{(units are in u)} \qquad \text{(Eq. 2-2b)}$$

Values for the mass defect of all known nuclei are tabulated in the Nuclear Wallet Cards (Sixth edition, 2000 , http://www.nndc/usndp/)

Exercise: Calculate the mass defect for the ^{4}He atom, given M(^{4}He) = 4.002603 u .

$$\Delta = (4.002603 - 4)\text{u}\ (931.494\ \text{MeV/u}) = 2.425\ \text{MeV}$$

It is important to stress that the chemical atomic weight listed in the Periodic Table is NOT the same as the mass of an individual isotope, unless the element has only one stable isotope; e.g. ^{27}Al. The **chemical atomic weight** is the average of all stable isotopes of a given element, $\langle M_Z \rangle$,

$$\langle M_Z \rangle = \Sigma\, f_i\, M_i\,, \qquad \text{(Eq. 2-3)}$$

where f_i is the relative abundance of each isotope of element Z (given in the Nuclear Wallet Cards) and M_i is the mass of each isotope (see Eq. 2-2b).

Example: Calculate the chemical atomic weight of copper, element 29.

f (^{63}Cu) = 69.09%; M(^{63}Cu) = 62.92959u
f(^{65}Cu) = 30.91%; M(^{65}Cu) = 64.92779u
<MCu> = (0.6909)(62.92959u) + (0.3091)(64.92779u) = **63.54u**

Finally, Einstein tells us that relativistic effects may be important for very high velocity particles; i.e. the total mass of a particle increases with its velocity, v. For example, a 200-MeV proton (rest mass M_o) accelerated at the IU Cyclotron Facility has a velocity of 0.6c, which increases its total mass by 25%, or $M/M_o = 1.25$. In this course we will use the classical equations of motion and assume relativity is negligible, except for a few special cases,

$$\text{Kinetic energy:} \quad E_K = M_o v^2/2 \qquad \text{(Eq. 2-4a)}$$

$$\text{Linear momentum: } p = M_o v \qquad \text{(Eq. 2-4b)}$$

These relationships are accurate to 1% for $v/c < 0.1$.

Nuclear Binding Energies

In discussing chemical stability the concept of electron binding energies plays a central role. Neon and sodium are adjacent elements in the Periodic Table, yet neon is a very stable inert gas while sodium is a highly reactive metal. This dramatic difference in behavior is directly related to the high binding energies of the valence electrons in neon compared to the low binding energy for the outermost electron in sodium. Briefly stated, high binding energies favor stability and low binding energies instability.

As a general definition, **BINDING ENERGY is the mass converted into energy that binds a system together**. This applies to both chemical and nuclear systems. In chemistry the mass change is so small that we don't think of it in $E = Mc^2$ terms. However, for nuclei, mass to energy conversion is a measurable quantity. Some examples of systems where mass is converted into binding energy are,

$$M(\text{ nucleus }) < (Z M_p + N M_n)$$
$$M(\text{ atom }) < M(\text{ nucleus }) + Z M_e$$
$$M(\text{ molecule}) < \Sigma M(\text{atoms })$$

In each case the difference in mass is the binding energy.

Total Binding Energy

In many respects the nucleus behaves like a nonpolar liquid drop. To examine the stability of such a nuclear system, it is useful to examine the **Total Binding Energy**, which is **the mass converted into energy when a nucleus is formed from its constituent nucleons,**

$$Z\,{}^{1}H + N\,{}^{1}n \rightarrow {}^{A}_{Z}X_{N} + \frac{TBE}{c^2}\ u\ . \qquad \text{(Eq. 2-5a)}$$

For which the mass balance equation is

$$TBE = [\, ZM_H + N M_n - M\left({}^{A}_{Z}X\right)]\, c^2\ \text{MeV}\ . \qquad \text{(Eq. 2-5b)}$$

Substituting Eq. 2-2b for M,

$$TBE = Z\Delta_H + N\Delta_n - \Delta\left({}^{A}_{Z}X\right)\ \text{MeV} \qquad \text{(Eq. 2-5c)}$$

The total binding energy is analogous to the heat of condensation of a liquid and the reverse equation corresponds to the heat of vaporization. The TBE for nuclei ranges from 2.2 MeV for the simplest nucleus, ^{2}H, to ~2000 MeV for uranium nuclei.

A more instructive quantity is the **average binding energy per nucleon, <BE>,** analogous to a molar heat of condensation (other than a factor of 6.023×10^{23}, to account for the number of particles in a mole instead of a single particle).

$$<BE> = TBE/A \qquad \text{(Eq. 2-6)}$$

The average binding energy is the amount of energy required to remove a nucleon from the nucleus, assuming all nucleons are equally bound (true for an ideal nonpolar liquid and to a fair approximation for nuclei).

Exercise: Calculate <BE> for ^{12}C.

$$6\,{}^{1}H + 6\,{}^{1}n \rightarrow {}^{12}C + TBE/c^2$$
$$TBE = 6\,\Delta_H + 6\,\Delta_n - \Delta({}^{12}C)$$
$$= 6\,(7.289) + 6\,(8.071) - 0 = 92.160\text{ MeV}$$
$$<BE> = 92.160/12 = \mathbf{7.680\ MeV}$$

One way to illustrate nuclear stability is in the form of a topological plot in which the average binding energy is equated with elevation in the proton versus neutron plane, as shown in the center frame of Fig. 2-1. In this allegorical description, most nuclei are unstable, forming a “sea of instability”. However, those nuclei with the highest average binding energies protrude from the sea to form a “peninsula of stability”. These are the nuclei we observe in Nature or can produce in the laboratory. Two relevant projections of the peninsula are:

(1) A sea level view – The elevation profile of the peninsula represents the average binding energy as a function of mass number A, as shown in the lower frame of fig. 2-1 (note suppressed zero on the y axis). Fig.2-2 shows a similar plot for the average binding energy of the lightest nuclei. The following features of the <BE> vs. A plot should be noted:

- To a first approximation the average binding energy is a constant value of ~ 8 MeV/nucleon, similar to the case for a nonpolar liquid drop. The lightest nuclei deviate from this pattern, showing much more pronounced structure in the average binding energies, so the analogy breaks down for nuclei with $A < 20$.
- ^{56}Fe is at the peak of the <BE> curve, making it the most stable nucleus in Nature. This fact plays an important role in stellar evolution.
- For nuclei lighter than ^{56}Fe, the average binding energy increases with mass number. Thus, when two light nuclei combine in a **nuclear fusion reaction**, more stable nuclei are formed and energy is released. Nuclear fusion reactions are the source of our Sun’s energy.
- For nuclei heavier than ^{56}Fe, the average binding energy decreases with increasing mass number. As a consequence energy is released when a heavy nucleus splits into two lighter nuclei in the **nuclear fission** process. Nuclear reactors derive their energy from the fission of ^{235}U.

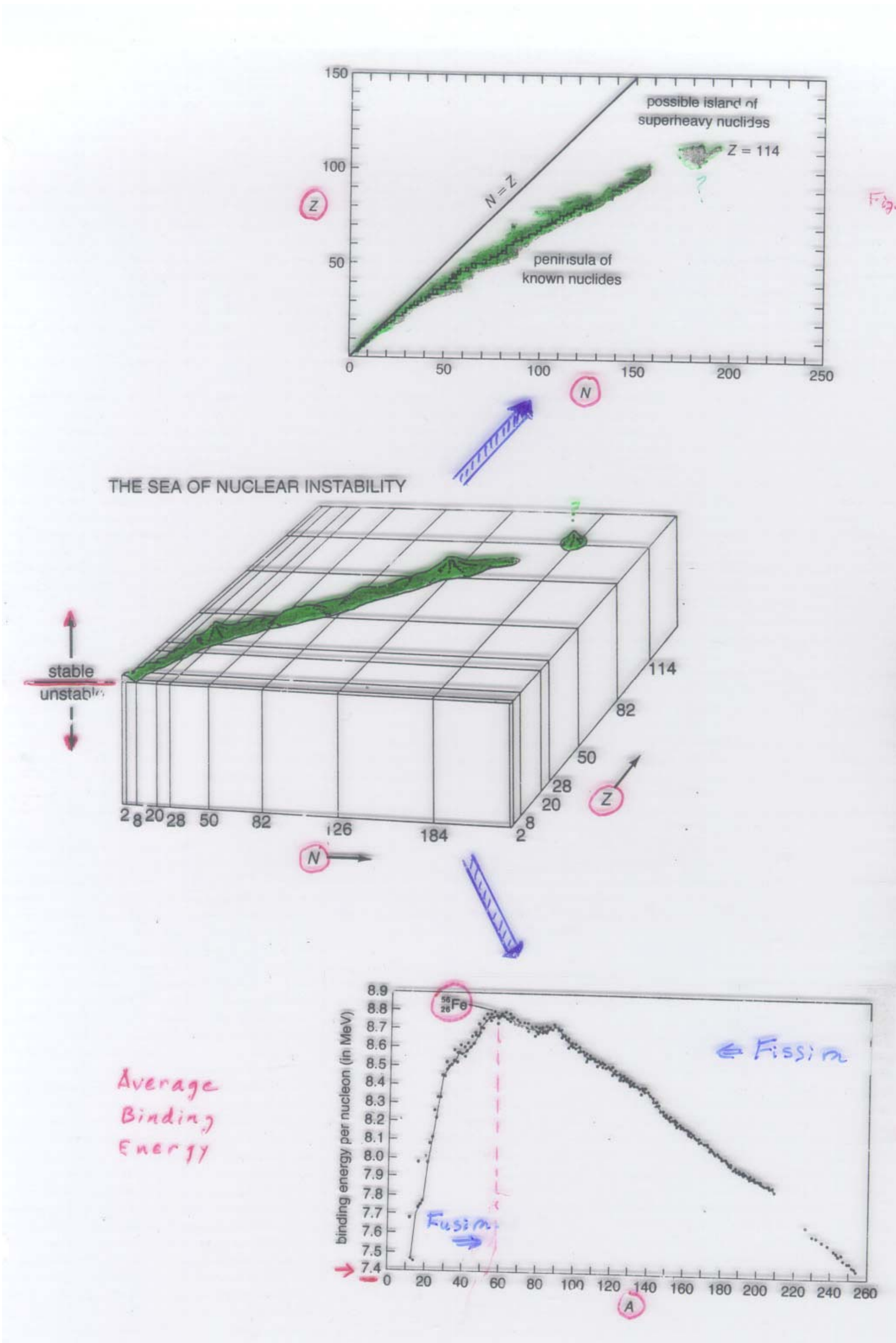

Fig. 2-1 Peninsula stability (center), with overhead (top) and sea-level (bottom) views.

- Fine structure is also observed in Figs. 2- 1 and 2-2, and is especially evident in the lightest nuclei, where quantum properties (shell structure) and even-odd effects (pairing) influence nuclear structure most strongly.

(2) A fly-over view – Looking down on the sea of instability from above (upper frame in Fig. 2-1) shows the location of the peninsula in the Z versus N plane. The most distinctive feature obtained from this view is that nuclei prefer to have roughly equal numbers of protons and neutrons; e.g. ^{12}C, ^{24}Mg, ^{40}Ca. As nuclei become heavier, they tend to have an excess of neutrons. In simple terms the neutron excess can be attributed to the need to minimize the repulsive effects due to the positive charge of the constituent protons.

Any successful model of nuclear structure must account for the features of nuclear stability shown in figs. 2-1 and 2-2.

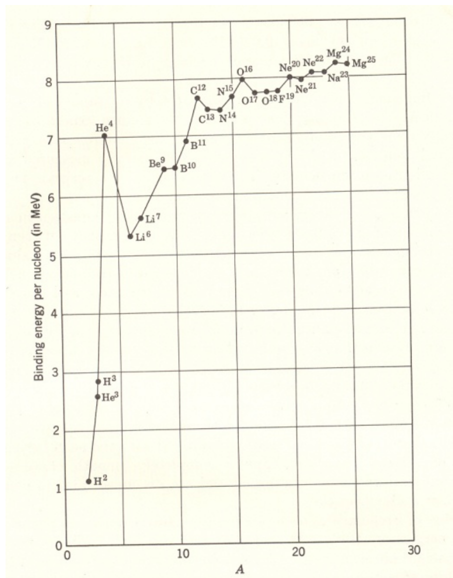

Fig. 2-2 Average binding energy curve for the lightest nuclei in the lower frame of Fig. 2.1.

Nuclear Energetics: Particle Binding Energies

What constrains the possible neutron-proton combinations that form the peninsula of stability in fig. 2-1? The answer to this question lies in the concept of particle binding energies, $\mathbf{B_i}$. **The particle binding energy is the energy required to remove a particle from the nucleus**. As the binding energy of a particle (proton, neutron, etc.) decreases, the nucleus becomes increasingly unstable toward spontaneous emission of that particle. This definition is comparable to the case for electron binding energies in an atom. (Note: in some presentations, binding energy, B_i , is defined as separation energy, S_i . Here we summarize the three most relevant cases:

- **Proton binding energy, $\mathbf{B_p}$** – the energy required to remove a proton from a nucleus, EQUATION: $${}^{A}_{Z}X + B_p c^2 \rightarrow {}^{A-1}_{Z-1}Y + {}^{1}_{1}H$$

(Eq.2-7)

CALCULATION: $B_p = \Delta(Y) + \Delta(H) - \Delta(X)$

- **Neutron binding energy, B_n** – the energy required to remove a neutron from a nucleus,

  EQUATION: $${}^{A}_{Z}X + B_n c^2 \rightarrow {}^{A-1}_{Z}X + {}^{1}_{0}n$$ (Eq.2.8)

  CALCULATION: $B_n = \Delta({}^{A-1}X) + \Delta n - \Delta({}^{A}X)$

- **Alpha-particle (^{4}He) binding energy , B_α --** the energy required to remove a ^{4}He from a nucleus,

  EQUATION: $${}^{A}_{Z}X + B_\alpha c^2 \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$ (Eq.2.9)

  CALCULATION: $B_\alpha = (\Delta Y) + \Delta(\alpha) - \Delta(X)$

In principle, one could calculate the binding energy for any particle: e.g. ^{2}H, ^{14}C, etc.

Example: Calculate the neutron binding energy for ^{12}Be.

$$^{12}Be + B_n c^2 \rightarrow {}^{11}Be + {}^{1}n$$

$$B_n = \Delta(^{11}Be) + \Delta(^{1}n) - \Delta(^{12}Be)$$

$$B_n = 20.174 + 8.071 - 25.007 = \mathbf{3.168\ MeV}$$

Nuclear Reaction Energetics: Q-values

In dealing with nuclear reactions, one would like to know whether a reaction is exothermic or endothermic, and by how much? For nuclear reactions we define a quantity Q, where **Q is the energy released in a nuclear reaction** between two colliding nuclei; i.e. for the reaction

$$A + B \rightarrow C + D + Q$$

$$Q = \sum \Delta \text{ (reactants)} - \sum \Delta \text{ (products)}$$ (Eq. 2-10)

As defined, the sign of Q is

Q = +, the reaction is EXOTHERMIC

Q = -, the reaction is ENDOTHERMIC.

[note: by analogy with chemical thermodynamics, Q = - ΔH]

Even if the Q-value is negative, an endothermic reaction can always be intitiated by accelerating one of the reactants and converting kinetic energy into mass energy.

Example: The nuclear fusion reactor utilizes the reaction

$$^{2}H + ^{3}H \rightarrow ^{4}He + ^{1}n + Q$$

$$Q = \Delta(^{2}H) + \Delta(^{3}H) - \Delta(^{4}He) - \Delta(^{1}n)$$

$$Q = 13.136 + 14.950 - 2.425 - 8.071 = \mathbf{17.590\ MeV}$$

The energy released in this reaction appears in the form of kinetic energy of the ^{4}He and the neutron, which can be converted into heat and subsequently electrical energy. Per gram of starting material, this reaction is one of the most efficient sources of nuclear energy in the universe.