SECTION 7: Nuclear Sizes and Shapes

Imagine you suddenly find yourself immersed in a dense fog in an unfamiliar location and need to explore your surroundings before attempting to move on. How can you learn the nature of your environment without being able to see anything, especially if you left your GPS at home? Conveniently, you happen to be carrying a bag of golf balls and being a science major, you know that you can perform a scattering experiment to learn about things you cannot see. As you start tossing golf balls randomly in all directions, you hear a splash (water over there), no sound at all, except after a very long time (definitely don't want to go that way), the thud of a ball striking wood (a house or a barn), the tinkle of broken glass (must be a house) and finally the plink-plink-plink of a ball bouncing on concrete (roadway ahead – proceed with caution). In essence, by scattering enough golf balls, you can map out your environment by using your ears as detectors to distinguish the wave lengths/frequencies of the sound wave emitted by the balls.

For microscopic objects such as atoms and nuclei, the same principle applies. However, the scattering probe needs to have a much shorter wavelength than sound in order to be comparable to the size of the object under investigation. In this case the proper metric is the wave length λ of a photon or particle, given by the DeBroglie wave length

$$\lambda_{photon} = \frac{hc}{E\gamma} = \frac{1.24\times10^{3}\text{fm}}{E\gamma(\text{MeV})} = \frac{h}{p} = \frac{h}{m\text{v}} = \frac{28.7\,fm}{\sqrt{A\bullet E}(\text{MeV})} = \lambda_{\text{particle}} \quad \text{(Eq.7.1)}$$

The classic applications of photon and particle scattering are the use of xrays to determine atomic sizes and Rutherford's alpha particle scattering experiment to deduce the size of the nucleus, respectively.

Table 7.1 lists characteristic energies (or velocities relative to c, the velocity of light) for scattering experiments with photons, electrons and nucleons in order to investigate atoms and nuclei.

Table 7.1 Relative energies for a given deBroglie wave length

probe	**Atoms(λ~10^{-8} cm)**	**Nuclei (λ~10^{-12} cm)**
photon	~ 10 keV (v ≡ c)	~ 100 MeV (v ≡ c)
electron	~ 100 eV (v ~ 0.1c)	~ 100 MeV (v ≈ c)
nucleon	~ 0.1 eV (v ~ 10^5 cm/s)	~ 10 MeV (v ~ 0.1 c)

From Table 7.1 it is apparent that smaller objects require higher energies (shorter wave length, $E_{photon} = hc/\lambda$).

Scattering using all the probes in Table 7.1 have been used to explore the sizes and shapes of nuclei. The end product of these experiments is most conveniently described in terms of a **nuclear density distribution, ρ (r)**. Fig. 7.1 illustrates density distributions for three objects: a hydrogen atom, a basketball and a uniform density sphere.

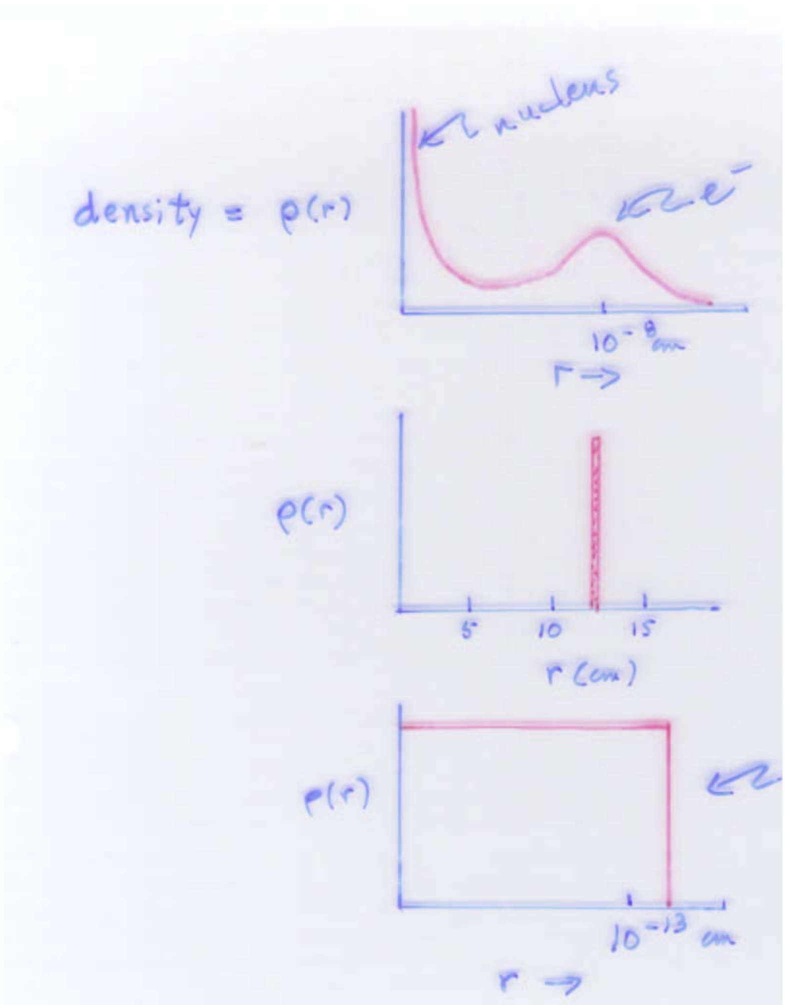

Fig. 7.1 Density distributions for a hydrogen atom (top), basketball (middle) and uniform density sphere (bottom).

A good first approximation to the density distribution of a heavy nucleus is that of a uniform density sphere (e.g. a solid rubber ball), as sketched in the bottom frame of Fig. 7.1. In this case the volume of the nucleus with A nucleons is

$$V_{nucle\mathbf{us}} = AV_{nucle\mathbf{on}} = (4/3)\pi R^3 ,$$

where $V_{nucle\mathbf{on}}$ is the volume of a nucleon (a constant) and R is the radius of a spherical nucleus. Assuming that the A nucleons are uniformly distributed throughout the sphere,

$$R = [3A\, V_{nucle\mathbf{on}} /4\pi]^{1/3} = \text{constant } A^{1/3} = r_o\, A^{1/3} . \qquad \text{(Eq. 7.2)}$$

The constant **r_o** is called the **nuclear radius parameter** and has a typical values of 1.2-1.4 fm, depending on the way in which the nuclear surface is defined, as discussed below.

Exercise: Calculate the radius of ^{216}Po (Z = 84) nucleus; $r_o = 1.40$ fm

$$R = r_o A^{1/3} = 1.40\text{fm}(216)^{1/3} = 8.40 \text{ fm} = 8.4 \times 10^{-13} \text{ cm}$$

The density distributions for real nuclei have been measured in scattering experiments with 21 GeV electrons ($\lambda \sim 0.1$ fm) at the Stanford Linear Accelerator (SLAC). Representative examples are shown in Fig. 7.2 .

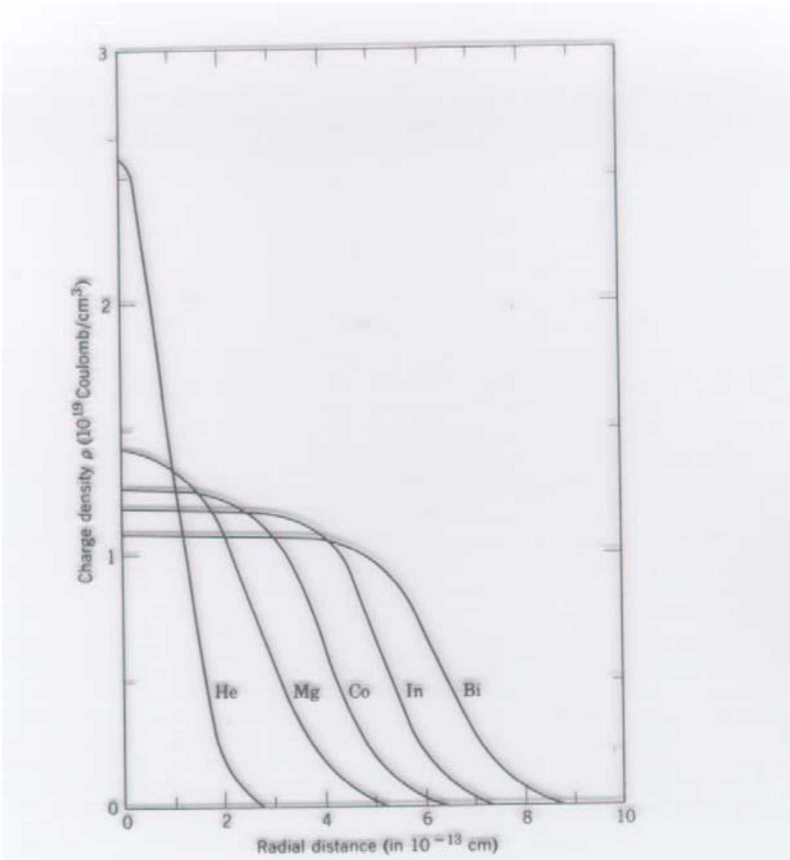

Fig 7.2 Charge distributions for several nuclei

For light nuclei, the uniform density sphere is seen to give a poor representation of real nuclei. But for heavier nuclei the approximation improves with increasing mass; i.e. heavy nuclei have a uniform central density, surrounded by a diffuse surface region (a cloudy crystal ball). The shape of the density distributions in Fig. 4.2 (Woods-Saxon shape) is described by a Fermi function, shown in Eq. 4.2.

$$\rho(r) = \frac{\rho_0}{1 + e^{-(r - R_{1/2})/d}} \qquad \text{(Eq. 4)}$$

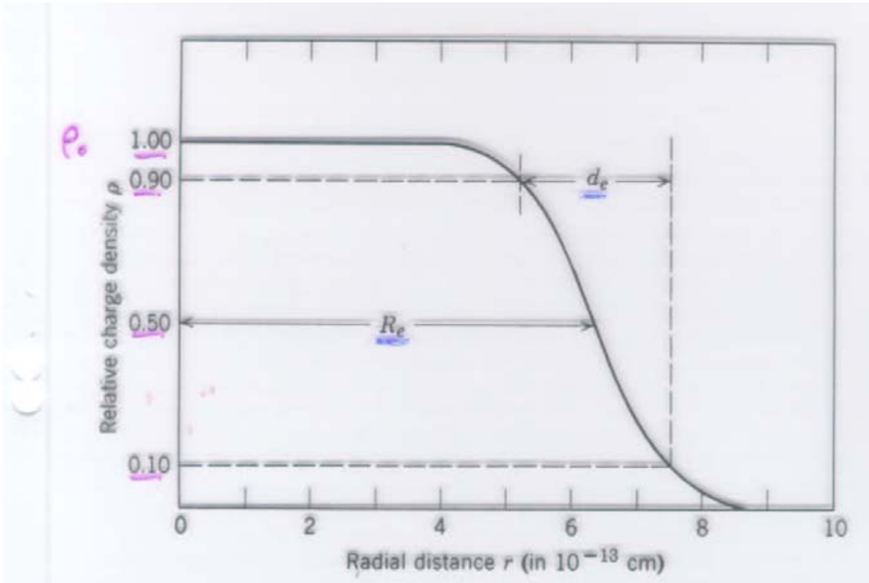

Fig. 7.3 Fermi function for a typical heavy nucleus

Here,

- ρ_o is the central nuclear density (~ 2 x 10^{14} g/cm^3); if the earth were a nucleus, its diameter would be about 20 meters;
- $R_{1/2}$ is the half-density radius $r_o A^{1/3}$ with r_o =1.07fm ; i.e. the radius at which $\rho/\rho_o = ½$;
- d is a diffuseness parameter that describes the fuzzy surface; it is the distance over which the density decreases form $0.9\rho_o$ to $0.1\rho_o$, typically a value of ~2.4fm for heavy nuclei.

Fig. 7.4 depicts several possible shapes for nuclei. Spherical shapes are generally found for nuclei with neutron or proton numbers near N or Z = 2, 8, 20, 28, 50, 82 and 126 (neutrons). These are the magic nucleon numbers, analogous to the values for the inert gases 2, 10, 18, 36, 54 and 86 that define the periodic table.. Recall that magic numbers occur when all the orbits in an atom or nucleus are filled, yielding a spherical configuration. However, not all nuclei have spherical nucleon distributions. Midway between the magic numbers, nuclei tend to become prolate spheroids (a > b =c, or rugby ball shapes) and exhibit rotational spectra, just as diatomic molecules.

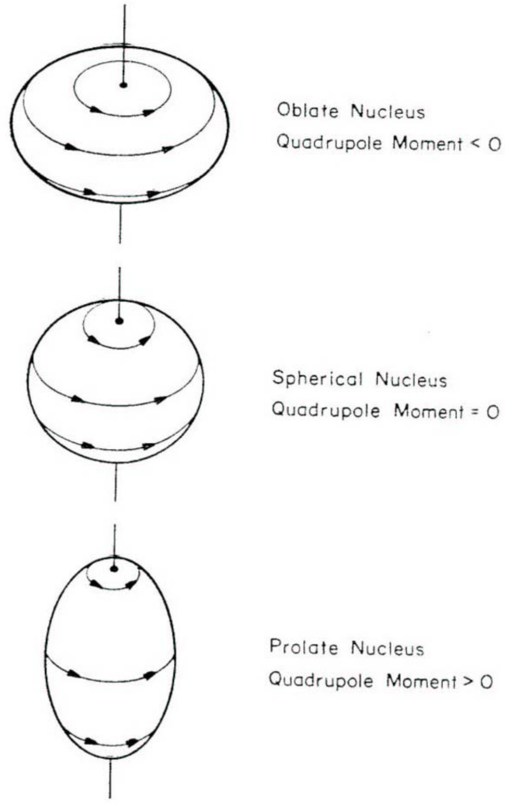

Fig. 7.4. Shapes of nuclei: oblate spheroid (top); spheroid (middle) and prolate spheroid (bottom).

In some excited nuclei, oblate shapes ($a < b = c$, or discus shaped) are observed. More exotic shapes have also been attributed to excited nuclei, for example octupole (pear-shaped), dumbell-like configurations in fission, and various oscillation modes in which protons and neutrons have different density distributions. The stable nuclei, however, tend to be either spherical or prolate spheroids.