

Section 4 : Accelerators

In addition to their critical role in the evolution of nuclear science, nuclear particle accelerators have become an essential tool in both industry and medicine. Table 4.1 summarizes the number of nuclear particle accelerators world-wide and the types of applications in which they are used.

Table 4.1

World wide inventory of accelerators, in total 15,000. The data have been collected by W. Scarf and W. Wieszczycka (See U. Amaldi Europhysics News, June 31, 2000)	
Category	Number
Ion implanters and surface modifications	7,000
Accelerators in industry	1,500
Accelerators in non-nuclear research	1,000
Radiotherapy	5,000
Medical isotopes production	200
Hadron therapy	20
Synchrotron radiation sources	70
Nuclear and particle physics research	110

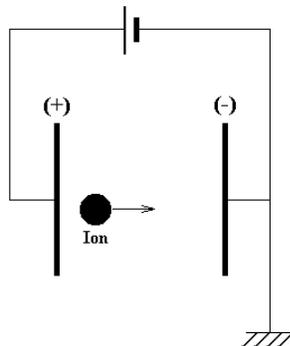
For a non-technical (no equations) discussion of accelerators, a good reference is <http://nobelprize.org/physics/articles/kullander/>. The principles of accelerator design fall into several categories, as discussed below.

Electrostatic Accelerators

Electrostatic devices operate on the principle of a constant electric field E , and are usually referred to as **Van de Graaf** or **Cockcroft-Walton** accelerators, in acknowledgement of their developers. These are high voltage accelerators that depend on accelerating ions through a large potential difference,

$$\Delta E = qe\Delta V, \quad (\text{Eq. 4.1})$$

where q is the atomic charge state of the ion, e is the electric charge and ΔV is the potential difference in Volts. Schematically, the operation is illustrated below:



The energy that can be achieved in an electrostatic device is limited by the maximum voltage that can be sustained by the electrostatic field gap before electric discharge occurs. A simple direct-current accelerator is diagrammed in Fig. 4.1.

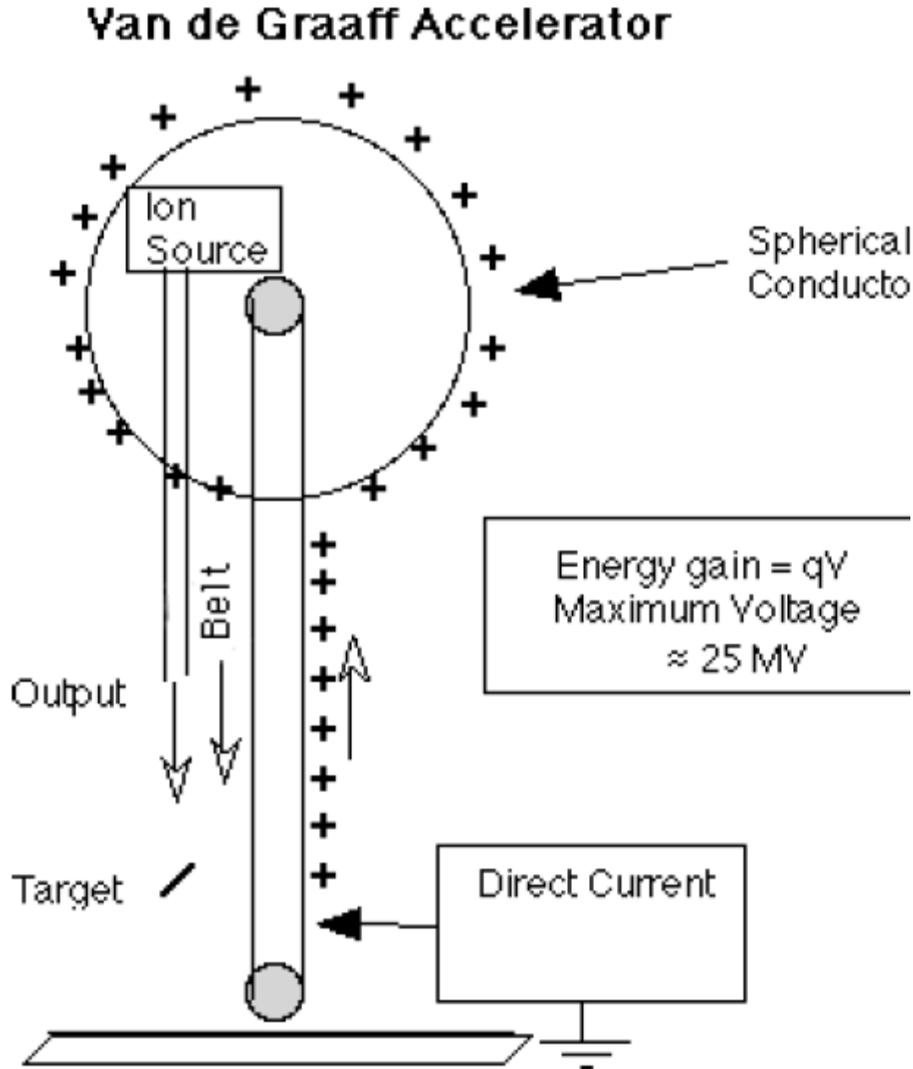
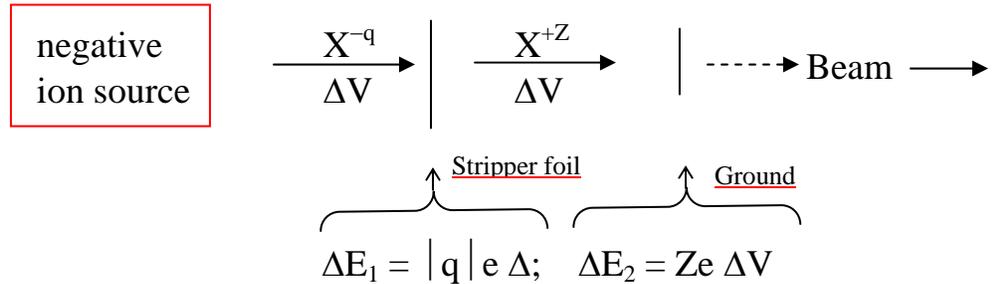


Fig. 4.1 Principle of operation of an electrostatic accelerator. Positive charge is stored on a spherical terminal and positive ions are repelled to ground through an evacuated tube.

In order to reach very high voltages, it is critical to insulate the positive and negative terminals from one another to prevent sparking. The largest terminal voltage that has been reached on any existing electrostatic accelerator is 25 MeV at the Oak Ridge National Laboratory Holifield facility, using SF₆ gas (which is very stable toward ionization) as an insulator.

In order to obtain higher ion energies, the concept of the **Tandem Van de Graaf** has been employed. The basic idea of the Tandem is to employ a two –step process to accelerate negative ions. Negative ions with charge state $-q$ are accelerated into a high voltage terminal with positive potential ΔV , reaching an energy $\Delta E_1 = |q| e \Delta V$. In the center of the terminal the ions are passed through a very thin stripping medium to create positive ions of charge $+Z$, which are then repelled by the positive potential to ground. In this way the ions receive two energy kicks from the same accelerator potential.



Thus the total energy gain is

$$\Delta E = \Delta E_1 + \Delta E_2 = (|q| + Z) e \Delta V \quad (\text{Eq. 4.2})$$

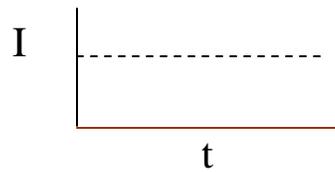
Example: A S^{-2} ion is accelerated in a tandem Van de Graaf with a terminal voltage of 25 MV. What is the maximum kinetic energy of the accelerated ion if the sulfur ion is fully stripped after passing through the stripping medium?

$$\Delta E = \{ |-2| + 16 \} 25 \text{ eMV} = 450 \text{ MeV}$$

Tandem accelerator at Brookhaven National Lab. (BNL)



Ion-source development has now successfully created negative ions for nearly all elements in the periodic chart. The major advantage of electrostatic accelerators is that they provide high precision beams. In addition, accelerator operation is relatively simple



compared to other types of accelerators. The time structure of the beam is continuous and beam currents I of the order of $10 \mu\text{A}$ can be obtained. The major drawback of the electrostatic devices is the limitation in maximum energy imposed by the maximum voltage that can be sustained by the terminal before sparking to ground. In addition to nuclear physics and astrophysics research, accelerators based on electrostatic principles find application in ^{14}C dating, ion implantation, radiation therapy, charged-particle activation analysis and accelerator mass spectrometry (AMS).

Cyclotrons

In elementary school we learned that if you wrap a wire around a nail and connect the wire to a battery, the circular motion of the electron current around the iron core creates a magnetic field. Conversely, if you place an electric charge in a magnetic field, it will follow a circular path. Cyclotrons operate on the principle that when a charged particle is placed in a constant magnetic field B , its **orbit time is independent of energy (the Cyclotron principle)**. This realization won E.O. Lawrence the Nobel prize in 1929. Thus, if you place an ion in a magnetic field and supply a boost of energy at some fixed point in its orbit, it will follow a spiral path as it increases in kinetic energy, as shown in Fig. 4.2.

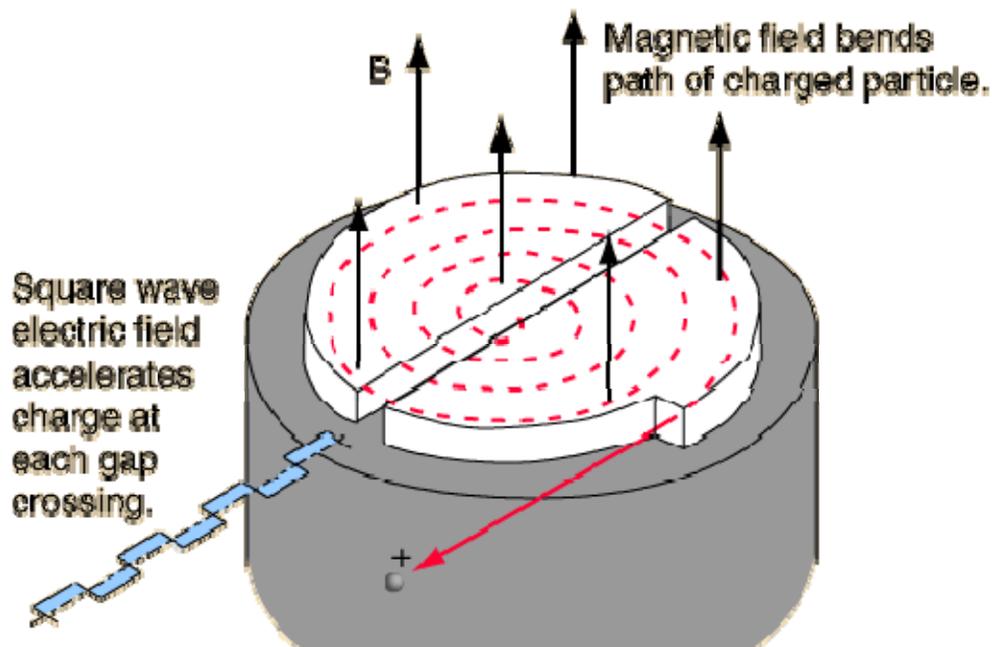
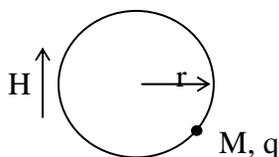


Fig. 4.2 Schematic diagram of the spiral motion of an ion in a magnetic field B . The ions are confined by two hollow, semicircular d-shaped electrodes and radiofrequency energy is supplied periodically in the gap region to boost the energy of the ions at each gap-crossing stage in their orbit.



The cyclotron principle is illustrated by the equations of motion for a non-relativistic particle of mass M and charge state qe in a magnetic field H of radius r .

The forces on the particle are the centripetal force exerted by the particle motion that is balanced by the containing force of the magnetic field:

$$\left\{ F_{\text{centripetal}} = \frac{Mv^2}{r} = F_{\text{magnetic}} = \frac{H v qe}{c} \right\} \quad (\text{Eq. 4.3})$$

Equating the two forces and solving for the ion orbit radius gives

$$r = \frac{Mv^2 c}{Hvqe} = \left(\frac{Mc}{Hqe} \right) v. \quad (\text{Eq. 4.4})$$

Since M , c , H and qe are constant, the radius depends only on the velocity of the ion. As long as the orbit velocity v is much less than the velocity of light, the orbit time becomes

$$t = \frac{2\pi r}{v} = \frac{2\pi Mcv}{v Hqe} = \frac{2\pi Mc}{Hqe}, \quad (\text{Eq. 4.5})$$

i.e. since all quantities on the right-hand side of Eq. 4.5 are constant, the orbit time is constant. This is the mathematical statement of the **cyclotron principle**. The orbit frequency is

$$\left\{ \omega = \frac{2\pi}{t} = \frac{Hqe}{Mc} \right\}. \quad (\text{Eq. 4.6})$$

Note that the frequency depends on the mass-to-charge ratio q/M of the ion, a principle that is applied in ion-cyclotron resonance studies (ECR) in the chemistry laboratory. For ions with $q/M \sim 1/2$ (e.g. ${}^4\text{He}^{+2}$, ${}^{12}\text{C}^{+6}$) in a magnetic field H of ~ 1.5 Tesla the orbit frequency is of the order of 10-30 MHz, or comparable to the low end of the FM radio frequency.

For each revolution in the cyclotron magnetic field an accelerating potential is applied to the ions, typically $\Delta E = qe \Delta V$, where $\Delta V \sim 50\text{-}250$ kV. In this way, the final energy of the ion is acquired in multiple small steps, instead of one or two big boosts as in the Van de Graaf accelerator. The maximum energy is limited only by the strength of the magnetic field, the magnet radius and the point at which the ion mass begins to increase due to relativistic effects (i.e. when $v/c \sim 0.1$) – in which case Eq. 4.5 breaks down. The

total energy is defined by the number of orbits n required to reach the maximum radius of the cyclotron r_{\max} :

$$\Delta E = n (qe) \Delta V . \quad (\text{Eq. 4.7})$$

Example: In order to reach the maximum radius in a given cyclotron with an accelerating potential of 200 kV, an ion of charge +2 requires 500 orbits. What is its maximum energy?

$$\Delta E = 500 (2e) 0.200 \text{ V} = 200 \text{ MeV}$$

For the general case the maximum cyclotron energy is given by

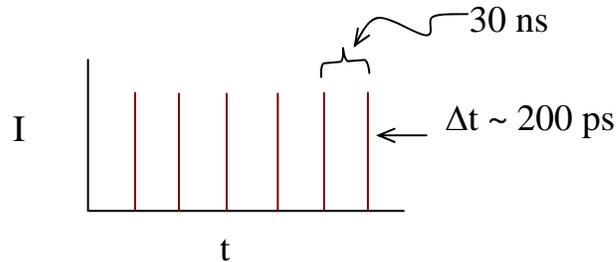
$$E_K = 1/2 M v^2 = 1/2 M \left(\frac{r^2 H^2 q^2 e^2}{M^2 C^2} \right) = \underbrace{\frac{1}{2} \left(\frac{r^2 H^2 e^2}{c^2} \right)}_{\text{cyclotron}} \underbrace{\left(\frac{q^2}{A} \right)}_{\text{ion}}, \quad (\text{Eq.4.8})$$

K

or $E_K = Kq^2/A$, for $v \ll c$. K is the figure of merit for a cyclotron's maximum energy. For example the superconducting cyclotron at Michigan State University has a value of $K = 1200$. Inserting the values for the constants we get:

$$E_K = 5.05 \times 10^{-3} H^2 r^2 (q^2/A) \text{ MeV/tesla}^2\text{-cm}^2 \quad (\text{Eq. 4.9})$$

Cyclotrons can accelerate most of the elements in the periodic table. They provide higher energy, but less precise beams than do Van de Graaf accelerators. By carefully structuring the magnetic pole pieces, the relativity constraint can be reduced, permitting higher energies. Intensities of $\sim 10 \mu\text{A}$ can be obtained, but in this case the time structure of the beam arrives in pulses, rather than being continuous.



For more historical information on cyclotrons, refer to the original paper Of Lawrence http://prola.aps.org/abstract/PR/v40/i1/p19_1 and in <http://www.aip.org/history/lawrence/> Facts about the IU cyclotron (IUCF), now used for proton therapy and radiation effects studies, are given in : <http://www.iucf.indiana.edu/whatis/facts.php>.

Synchrotrons

Both Van de Graaf generators and cyclotrons are limited in the maximum energy they can achieve, the former by the maximum breakdown voltage it can maintain and the latter

by relativistic effects. In order to probe the structure of the nucleon itself, higher energies are required, which has led to the development of the **synchrotron**. Instead of maintaining a constant magnetic field and variable particle radius as in the cyclotron, the synchrotron uses the opposite approach – a variable magnetic field and a fixed radius. Fig. 4.3 shows the basic components of a synchrotron.

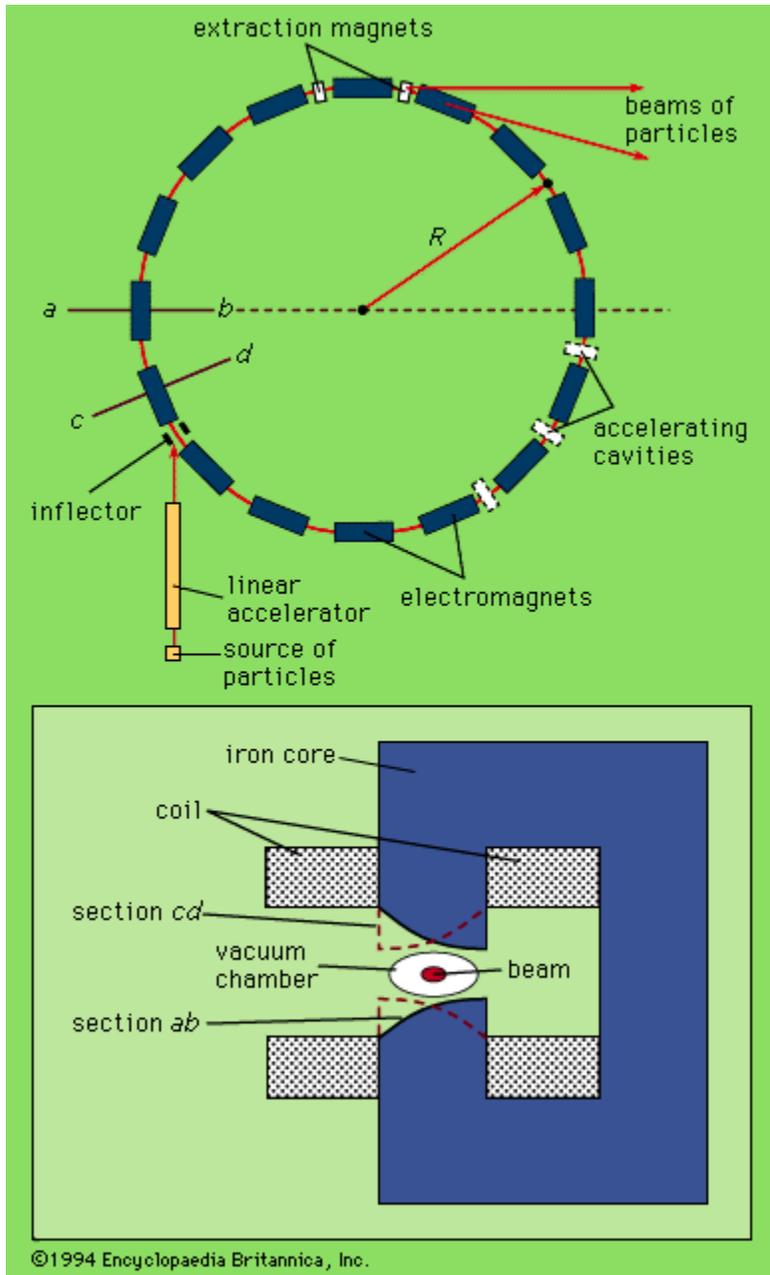


Fig. 4.3 Upper frame: diagram showing the injector, ring, bending magnets, accelerating cavities and extraction magnets. Lower frame: cross-sectional cut through one of the bending magnets, showing the location of the beam as it passes through the ring vacuum chamber and the magnetic coils and iron core.

An ion beam is first injected into the ring with a smaller accelerator, for example a Van de Graaf or more commonly a linear accelerator (described below). The beam trajectory is controlled by a series of bending magnets and the beam is accelerated by rf energy in various cavities. As the energy increases, the magnetic field is ramped up correspondingly, thus keeping the beam of particles in the correct orbit inside the synchrotron ring. The process is computer-controlled and allows for the relativistic increase in mass of the ions as they reach higher and higher energies. In this way the only limitations on maximum energy are the size of the ring and the strength of the bending magnets. The two highest-energy synchrotrons in the world today, the CERN LHC in Switzerland and the Fermilab Tevatron in Illinois, which have ring diameters of 4500m and 1000m, respectively, corresponding to maximum energies of 7 TeV and 2 TeV (1 TeV = 10^{12} eV). At these energies the velocity of the ions is $v/c \sim 0.999$.

Synchrotrons are used primarily to accelerate (1) the simplest particles for probing the structure of the nucleon and (2) electrons for use as light sources; i.e. the bending of the electron beam in the ring creates energetic photon beams that are used in biochemical and materials research. These latter accelerators are called **Light Sources**. However, beams of heavy ions up to uranium have been produced by some accelerators, most recently by the RHIC (Relativistic Heavy Ion Collider) at Brookhaven National Laboratory in New York. For more on synchrotrons see <http://accelconf.web.cern.ch/AccelConf/e96/PAPERS/ORALS/FRX04A.PDF>

Linear Accelerators

The basic concept of the **linear accelerator** (linac) is to provide multiple energy boosts along a linear path. Schematically they are analogous to a series of electrostatic devices, except that they use lower voltages and obtain higher energies by giving multiple kicks to the ion injected into the machine. Thus the total energy one can achieve in a linear accelerator depends on the number of energy boosts n the ion receives in passing through a series of n drift tubes, as diagrammed in Fig. 4.4.

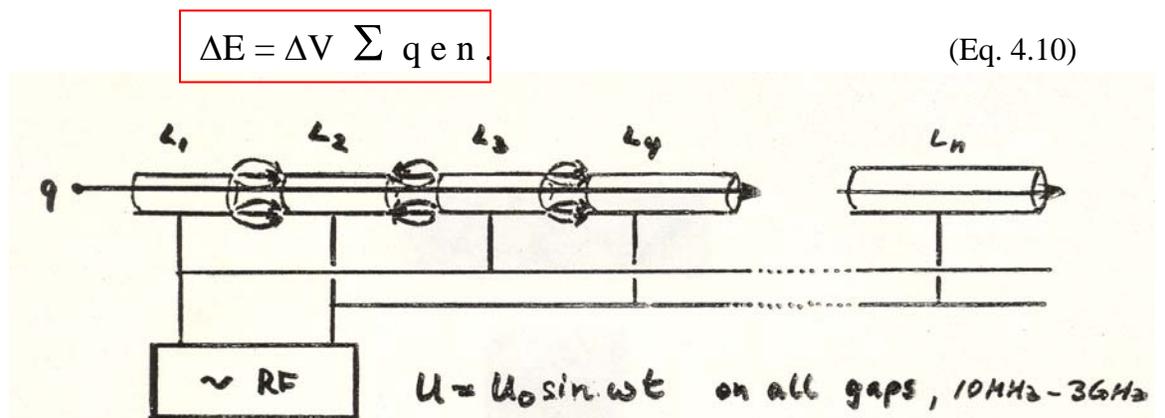


Fig 4.4 Schematic diagram of several stages of a linear accelerator in which the electric field U oscillates with frequency ω along the path of multiple drift tubes..

Ions are injected into the machine and feel the accelerating potential of the first drift tube. Once inside the drift tube the ion is in a field-free region and receives no acceleration. The voltage on the tubes is varied at a frequency that insures that as the particle moves between each pair of tubes, it sees the attractive potential of the next drift tube, thus receiving an additional energy kick.

Inside the drift tubes the voltage U is the same; i.e. no acceleration occurs. The voltage on the tubes is applied at a radiofrequency that insures that as a particle enters the gap between tubes, it experiences an acceleration. Since all the drift tubes are pulsed at the same frequency, each drift tube must be longer so that the particles reach the gap at the same time. The drift tube length L is written

$$L = V \frac{T}{2} = \beta \frac{\lambda}{2}, \quad (\text{Eq. 4.11})$$

where $\beta = \frac{V}{c}$ and T is the period. After n gaps,

$$\frac{1}{2} M V_n^2 = n(q)U_0.$$

Rearranging,

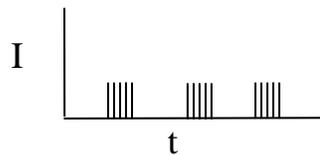
$$V_n = \left(n \frac{2qU_0}{M} \right)^{\frac{1}{2}}$$

$$L_n = \left(n \frac{2qU_0}{M} \right)^{\frac{1}{2}} \frac{T}{2} \quad (\text{Eq. 4.12})$$

Notice that the drift tubes have to get longer (greater n) as the particle accelerates so that the particles always reach the gap at the same time.

Linear accelerators serve a wide variety of nuclear science applications. The highest energy machine is the Stanford Linear Accelerator (SLAC) in California, which produces 25-GeV electron beams for high energy physics research. SLAC's two-mile length required that the curvature of the earth be taken into account in its construction in order to insure a linear path for its highly precise beams. Linear accelerators are frequently used as injectors to higher energy machines such as RHIC and LPS. At lower energies linear accelerators that employ superconducting rf cavities are used for nuclear science studies with heavy-ion beams. Both electron and proton beams based on the linear principle are widely employed for medical therapy.

Very high intensity beams of up to 1mA can be attained in linacs. The machines operate as pulsed machines with a time structure shown schematically below.



Most modern nuclear research facilities today couple two or more accelerators components to achieve the goals of the experimentalists who employ them. The Indiana University Cyclotron (IUCF) is one such facility, using a linear accelerator and an injector into a cyclotron to achieve beams of 200 MeV protons.



IUCF Cyclotron