SECTION 10: Alpha Decay

Alpha decay involves the emission of a ^{4}He nucleus from a heavy element, represented by the generic equation

$$^{A}_{Z}X \rightarrow {}^{4}_{2}He + {}^{A-4}_{Z-2}Y + Q_{\alpha} \quad . \qquad \text{(Eq. 6-1)}$$

Note that the chemical states of the products of this reaction involve a 2+ charged He ion and a 2- charged heavy residue. This can lead to some unusual chemistry, for example, the inert gas atom radon, formed in the alpha decay of radium, would be in a Rn^{2-} state.

The dynamic range of alpha decay half-lives is one of the largest of any observed physical phenomenon in Nature. Measured half-lives extend over a factor of 10^{44}, from as long as 10^{16} years to 10^{-21} seconds. Alpha-particle decay rates depend very strongly on the energetic factor Q_{α}, which tends to be positive (thermodynamically allowed) only for those nuclei with A>140.

<u>Energetics</u>

An example of alpha decay is given by the following reaction:

$$^{228}_{90}Th \rightarrow {}^{224}_{88}Ra + {}^{4}_{2}He + Q_{\alpha}$$

Exercise: what is the Q-value for the above reaction?

$$Q_{\alpha} = \Delta(^{228}Th) - \Delta(^{224}Ra) - \Delta\alpha$$
$$= 26.758 - 18.313 - 2.425 = 5.520 \text{ MeV}$$

The measured energy of the alpha particles emitted from ^{228}Th is 5.423 MeV. Why is this different from the Q-value? To explain this difference, one must remember that **both energy and momentum must be conserved** in this two-body breakup process; i.e.

Momentum Conservation: $0 = \vec{p}_{\alpha} + \vec{p}_{R} \quad \left(p = \sqrt{2ME}\right)$ (Eq.10-2)

Energy Conservation: $Q_{\alpha} = E_{\alpha} + E_{R} \qquad (E = 1/2\, Mv^{2})$ (Eq.6-3)

From these two equations, it is apparent that the missing energy is carried away by the heavy recoil nucleus, in this case, ^{224}Ra. Solving equations 10.2 and 10.3, we obtain the general equations for the alpha and recoil energies for any given decay,

$$E_{\alpha} = \frac{M_R}{M_R + M_{\alpha}} \bullet Q_{\alpha} \approx \frac{A_R}{A_R + A_{\alpha}} \bullet Q_{\alpha} \qquad \text{(Eq. 10-4)}$$

$$E_{R} = \frac{M_{\alpha}}{M_R + M_{\alpha}} \bullet Q_{\alpha} \approx \frac{A_{\alpha}}{A_R + A_{\alpha}} \bullet Q_{\alpha} . \qquad \text{(Eq.10-5)}$$

Note that for convenience, the particle mass has been replaced by its mass number. Since we are dealing with ratios, this facilitates calculations and introduces little error. These equations tell us that the alpha particle kinetic energy must be a single value. This statement is true for any two-body breakup process. Applying Eqs. 10.4 and 10.5 to the ^{228}Th decay above,

E_{α} = (224/228) 5.520 = **5.423 MeV,** and
ER = (4/228) 5.520 = **0.097 MeV.**

The measured kinetic energy spectrum for alpha particles emitted in the decay of ^{228}Th is shown in Fig 10.1 These energies have been translated into a decay scheme in Fig. 10.2.

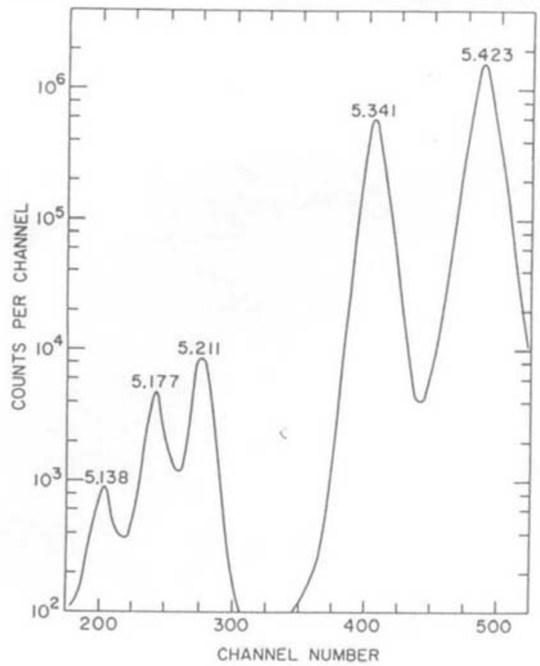

Fig. 10.1 Kinetic energy spectrum of alpha particles emitted from ^{228Th}.

In addition to the 5.423 MeV alpha particles predicted from the Q-value, several additional alpha particle groups are also observed. The explanation for this behavior lies in the energy level structure of ^{224}Ra, as shown in the bottom frame of Fig. 6.2. In some

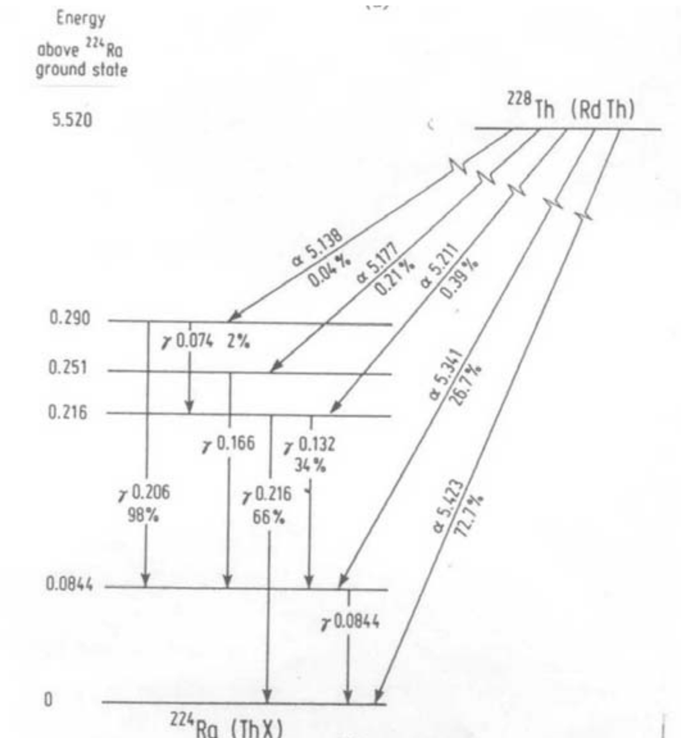

Fig. 6.2 Level scheme for alpha decay of ^{228}Th. Note: the alphaenergies on the plot are kinetic energies, not Q_α values. The Q-values are listed on the left of the diagram.

fraction of the decays, given by the percentages associated with each alpha energy on the figure, the decay occurs to an excited energy state of ^{224}Ra. The fractional decay probability is called a **branching ratio**. In all cases the total energy going from ^{228}Th to ^{224}Ra is the same; i.e. Q_α . When decay occurs to an excited state, energy and momentum balance result in the following expression for the alpha particle energy

$$E'_\alpha = \frac{A_R}{A_R + A_\alpha} \bullet (Q_\alpha - E_\gamma) \text{ and } E'_R = \frac{A_\alpha}{A_R + A_\alpha} \bullet (Q_\alpha - E_\gamma) \quad \text{(Eq. 10-6)}$$

where E_γ is the energy of the gamma ray that is emitted from the excited state. In this derivation, the momentum of the recoil nucleus that accompanies gamma decay is ignored, since the recoil energy is very small for the massless gamma ray and it is safe to assume that $E_\gamma = Q_\gamma$ to a good approximation. The gamma ray recoil momentum is difficult to detect, but can be observed in the Mossbauer effect.

Remember, regardless of which decay path is followed, the total energy must be equal to

$$\mathbf{Q_\alpha = E_\alpha + E_R = E_\alpha' + E_R' + E_\gamma}$$

Measured Q-values range from as low as 2 MeV for samarium (Z =62) to as high as 11.7 MeV for decay from an excited state of ^{212m}Po. Most alpha-decay Q-values are in the range 4 – 8 MeV.

Why do heavy elements emit alpha particles rather than other particles, e.g. protons, neutrons or heavier particles? One answer to this question can be determined by examining the Q-values for proton and neutron decay. Consider the case of ^{238}U:

$$^{238}U \rightarrow {}^{234}Th + {}^{4}He + Q_\alpha \;; \quad Q_\alpha = \Delta(^{238}U) - \Delta(^{234}Th) - \Delta(^{4}He) = +4.27 \text{ MeV}.$$
$$^{238}U \rightarrow {}^{237}Pa + {}^{1}H + Q_p; \quad Q_p = \Delta(^{238}U) - \Delta(^{237}Pa) - \Delta(^{1}H) = -6.151 \text{ MeV}$$
$$^{238}U \rightarrow {}^{237}U + {}^{1}n + Q_n\;; \quad Q_n = \Delta(^{238}U) - \Delta(^{237}U) - \Delta(^{1}n) = -7.584 \text{ MeV}.$$

That is, the Q-values for proton and neutron decay are negative (their binding energies are positive) and therefore decay is thermodynamically prohibited.

However, there are other cases where the Q-value is positive but decay does not occur, or at least is very rare, for example ^{14}C and ^{18}O. To understand decay lifetimes and the preference for alpha decay via these alternative modes, it is necessary to examine the schematic potential-energy diagram for a heavy element as a function of distance, as illustrated in Fig. 10.3.

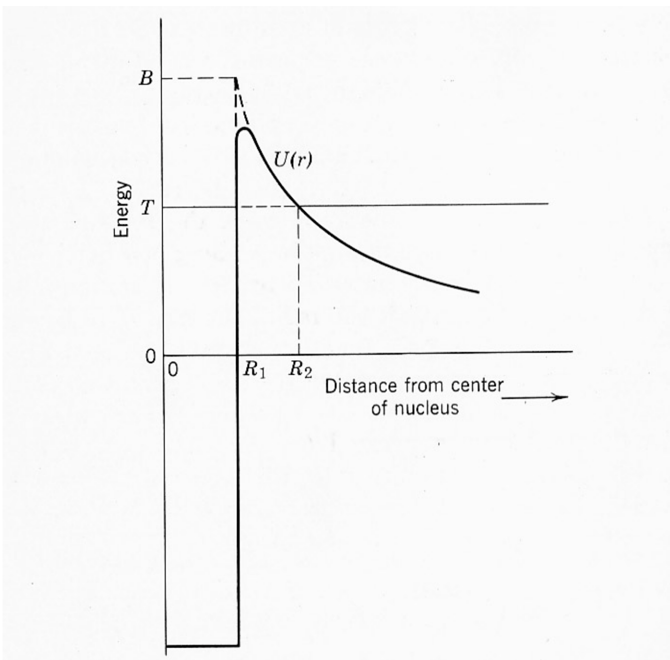

Fig. 10.3 Potential energy diagram for a nucleus, assuming a square-well potential.

Consider the situation as a charged particle of kinetic energy T approaches a nucleus from a large distance. The two distinguishing features of Fig. 10.3 are (1) the strongly attractive region created by the attractive nuclear force for $d < R_1$ (approximated by a square well potential) and (2) the repulsive component for $d > R_1$ due to the electrostatic repulsion between the two positively-charged nuclei, governed by Coulomb's Law. Thus the total potential energy is

$$V(d) = V_{nuclear}(\text{attractive}) + V_{Coulomb}(\text{repulsive}),$$

where the Coulomb potential $V_{Coulomb}$ is

$$V_{Coulomb} = Z_1 e \, Z_2 e/R = 1.44 \, Z_1 \, Z_2 /R \text{ MeV-fm} \qquad \text{(Eq. 10.7)}$$

$$R = r_o(\, A_1^{1/3} + A_2^{1/3}); \quad r_o \sim 1.40 \text{ fm.}$$

The point at which V(d) is a maximum (d = R_1) corresponds to touching spheres and is called the **Coulomb Barrier**.

Exercise: What is the Coulomb barrier encountered by an alpha particle approaching a ^{240}Pu nucleus (Z = 94)?

$$V_{Coulomb} = 1.44(2)(94)/(1.40)(4^{1/3} + 240^{1/3}) = 24.8 \text{ MeV}$$

Therefore, in this classical situation, the alpha particle must have at least 24.8 MeV of kinetic energy to penetrate the nucleus (actually slightly more in order to conserve momentum).

Now consider the reverse case of an alpha particle that forms inside a ^{244}Cm nucleus (Z = 96) and wants to escape, as sketched below in Fig. 10.4. The Q-value for this decay is 5.902 MeV, much lower than the 24.8 MeV Coulomb barrier. Thus, classically the alpha particle cannot get out, despite the fact that the decay is thermodynamically allowed.

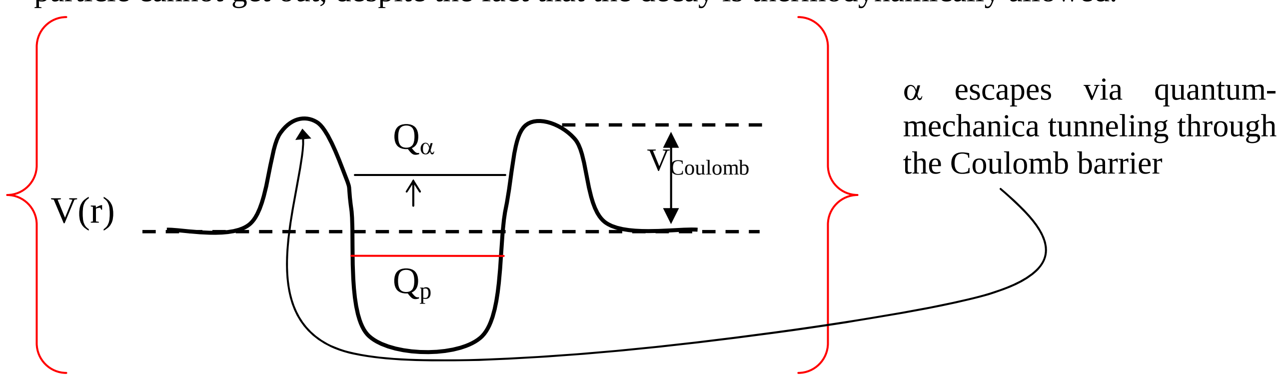

Fig. 10.4 Sketch of the potential energy surface encountered by an alpha particle and a proton inside the nucleus

So why does ^{244}Cm decay with a half-life of 18.1 years? In the late 1920s George Gamov explained alpha decay as a quantum mechanics tunneling problem, one of the early triumphs of quantum theory. Quantum mechanics (greatly simplified here) predicts that the probability for the alpha particle to tunnel through the Coulomb barrier, P_α ,can be expressed as

$$P_\alpha \sim P_{formation} \, e^{(Q_\alpha - V_{Coulomb})} \sim 1/t_{1/2}(\alpha) \qquad \text{(Eq.10.8)}$$

Where $P_{formation}$ is the probability for formation of an alpha particle inside the nucleus, a slowly varying function. Thus, to a first approximation one can write

$$\log \, t_{1/2}(\alpha) \sim V_{Coulomb} - Q_\alpha \, , \qquad \text{(Eq. 10.9)}$$

which predicts an exponential dependence of alpha decay half lives on Q-value and accounts for the 10^{44} range of observed lifetimes. In Figs 10.5 and 10.6 are shown plots of alpha decay Q-values as a function of mass number and Q_α versus log $t_{1/2}(\alpha)$, respectively. Systematic behavior such as that shown here is a valuable tool for

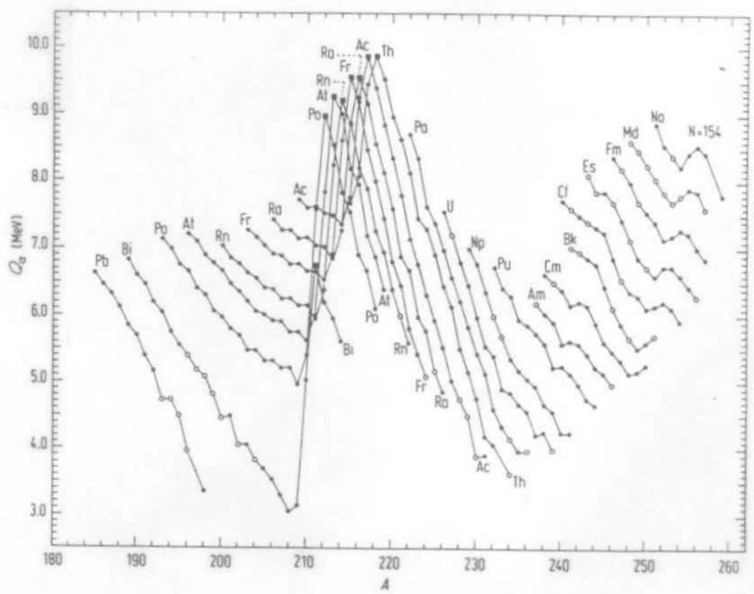

Fig. 10.5 Alpha decay Q-values as a function of mass number

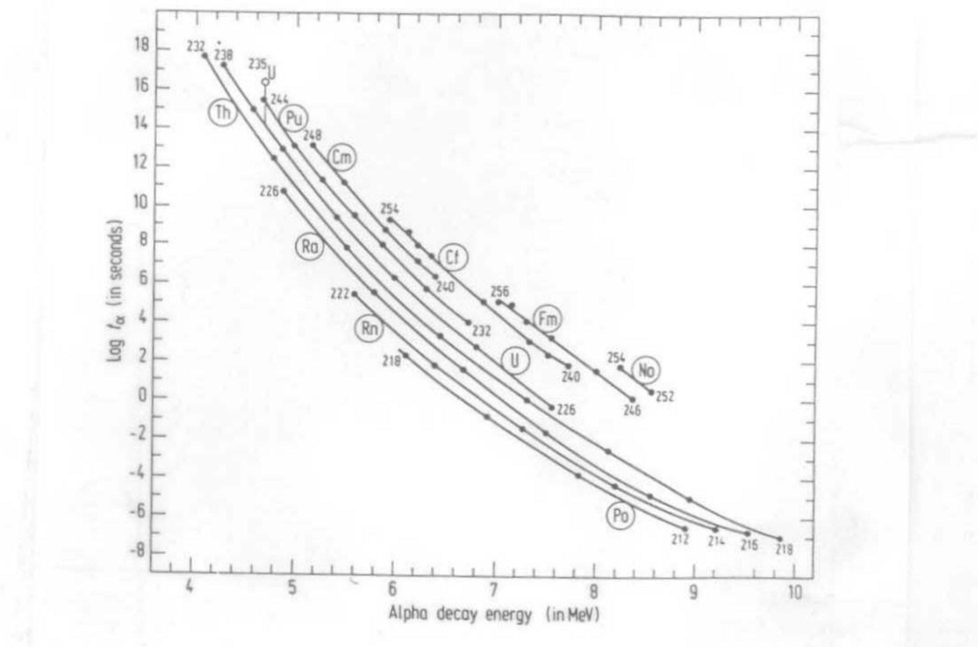

Fig. 10.6 Alpha decay Q-value as a function of log $t_{1/2}(\alpha)$

predicting the properties of unknown heavy elements. The discontinuity in Q_α values near A ~ 210-220 is a consequence of nuclear shell structure.

The Coulomb Barrier explains why the decay of nuclei heavier than ^4He is rare. From examination of Eq. 10.7, it is clear that the Coulomb Barrier increases nearly proportionally with the Z of the emitted particle, so that for Z = 6, the barrier is nearly three times higher than for He^4 , thus greatly inhibiting the tunneling probability.