

What did you learn in the last lecture?

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Beta stability, the LD Mass Formula, and Accelerators

Simplest form of LD Mass Formula

$$\begin{aligned} \text{TBE} &= C_1 A - C_2 A^{2/3} - C_3 Z^2/A^{1/3} - C_4 (N-Z)^2/A^2 + C_6 \delta/A^{1/2} \\ \langle \text{BE} \rangle &= C_1 - C_2 A^{-1/3} - C_3 Z^2/A^{4/3} - C_4 (N-Z)^2/A^3 + C_6 \delta/A^{3/2} \end{aligned}$$

Line of Beta Stability – Isobars

1. Beta Decay – Form of Radioactive Decay

$n \leftrightarrow p$ conversion inside nucleus

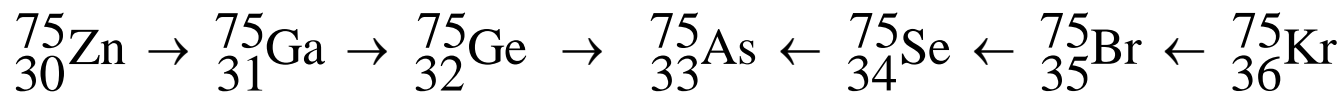
\therefore **A** doesn't change; just N/Z ratio – ISO BARS

Most probable N/Z ratio \equiv Line of Beta Stability

2. Example: **A = 75** chain

N/Z = 1.5

N/Z = 1.08



$n \rightarrow p$



Beta-
Stable
Nucleus

$n \leftarrow p$

3. Isobaric Mass Formula

Since in β decay the mass number remains constant, it is useful to develop an isobaric mass formula (“iso” means same in Greek).

- A is constant in Binding Energy Equation (& $\langle BE \rangle$ form)

$$\left(\Delta \frac{A}{Z} X \right) = Z \Delta_H + N \Delta_n - TBE$$

plug in LD Mass Equation

- $\left(\Delta \frac{A}{Z} X \right) = d_1 Z^2 + d_2 Z + d_3 + d + d_4 \delta$, where $d_i = f(C_i, A)$
- What is the functional form of this equation? minimum defines most stable nuclide for a given A. These values define the "valley of stability".
Atomic number of this nucleus is Z_A .

Valley (line) of Beta stability

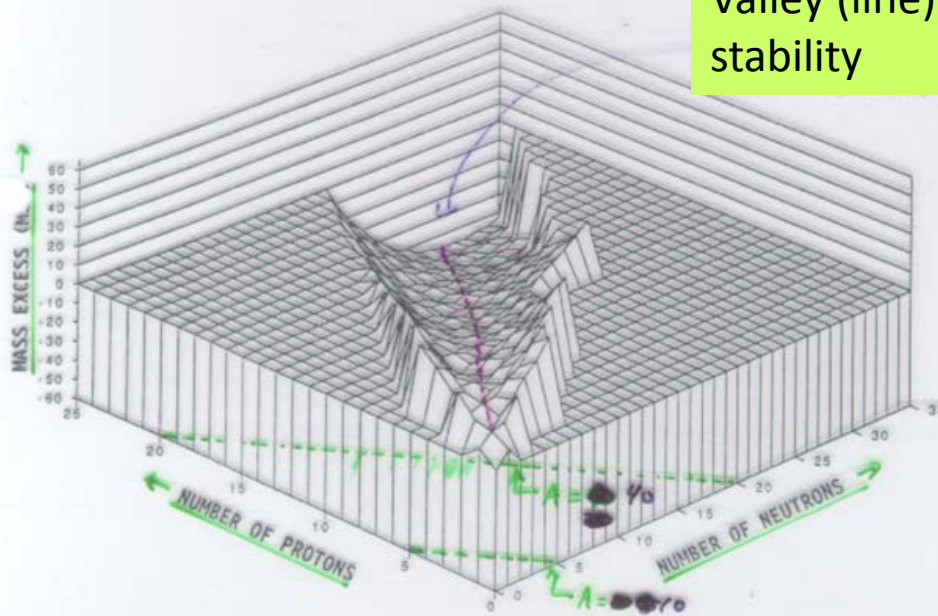


Figure 3.8. A three-dimensional representation of the valley of beta stability

Two Cases

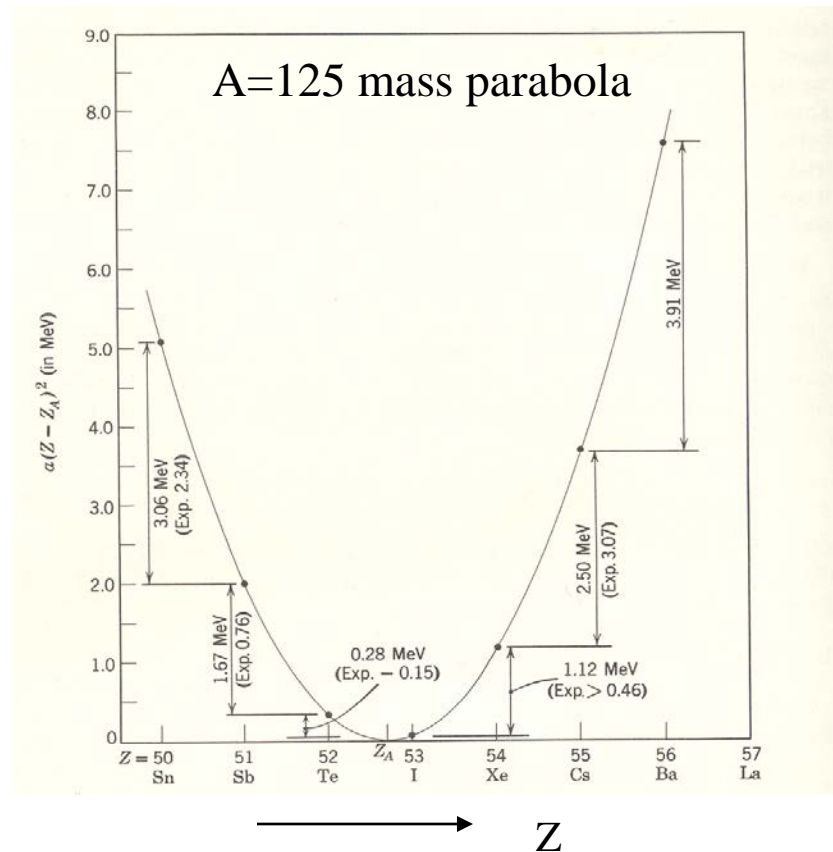
a. Case I: odd-A nuclei ($\delta = 0$)

- Single parabola: $\Delta\left(\frac{A}{Z}X\right) = d_1 Z^2 + d_2 Z + d_3$; one Parabola

- Most probable charge: $\frac{\partial \Delta\left(\frac{A}{Z}X\right)}{\partial Z} = 2d_1 Z_A + d_2$

RESULT: ONE STABLE ISOTOPE PER MASS NUMBER

$M(Z) - M(Z_A)$
in MeV



Consider the two mass parabolas of $A=75$ and $A=157$. What do you notice?

$$M(Z) - M(Z_A)$$

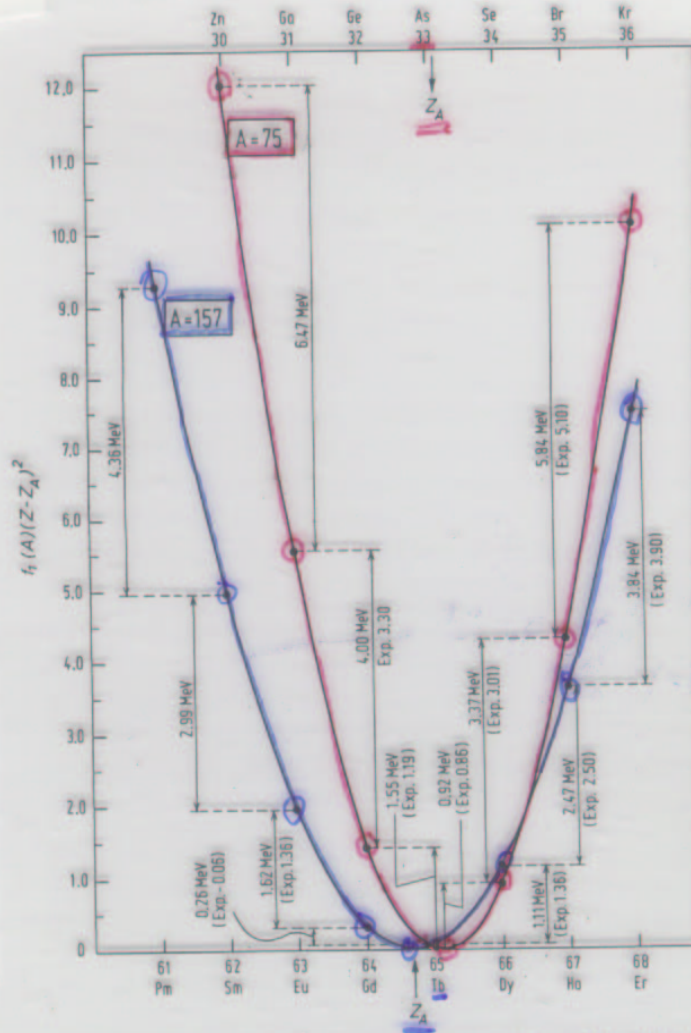
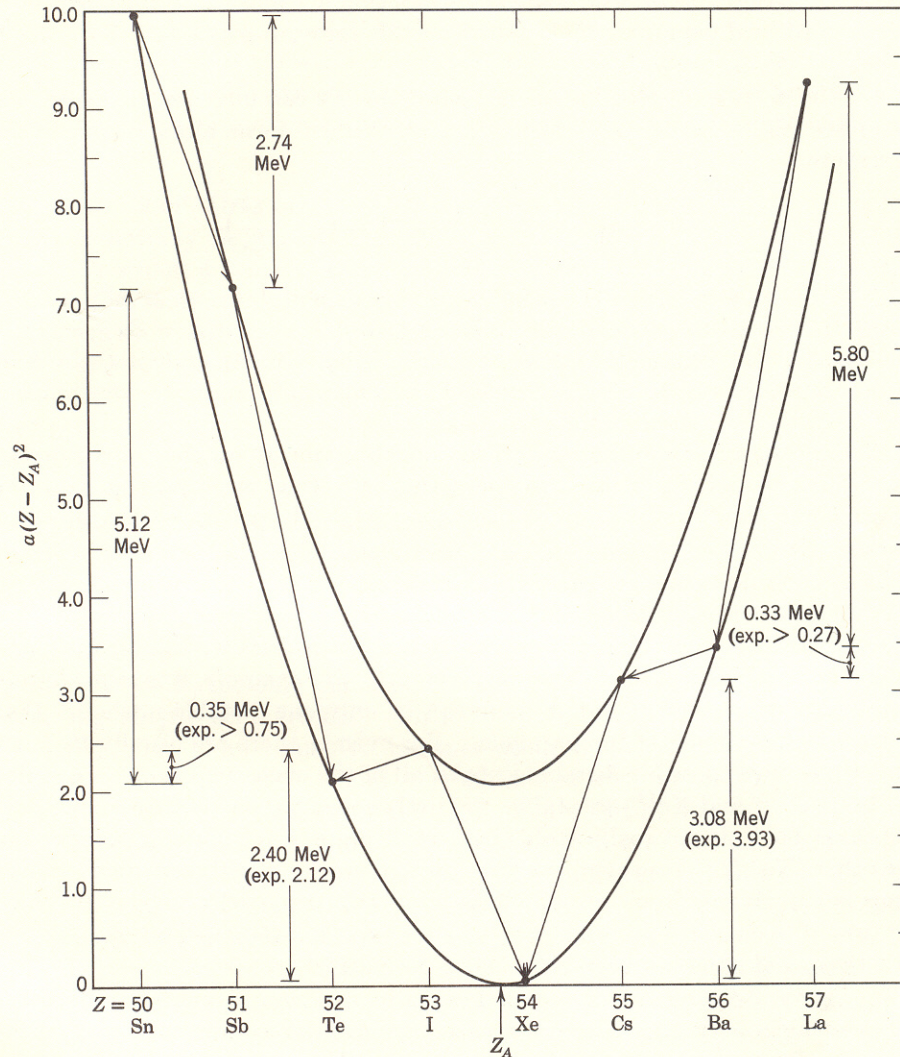


Fig. 2-7 Mass parabolas for $A = 75$ and $A = 157$, as calculated from (2-8). Calculated mass differences between neighboring isobars are indicated, with experimentally determined values shown in parentheses for comparison. The top Z scale refers to $A = 75$, the bottom one to $A = 157$.

b. Case II: even A nuclei ($\delta = \pm 1$)

$$(\Delta \frac{A}{2} X) = d_1 Z^2 + d_2 Z + d_3 \pm \delta d_4$$

RESULT: Two parabolae ; even-Z always lower CAN HAVE 1,2, OR 3 STABLE NUCLEI PER A.



- $A=128$
- Upper parabola is odd-odd
- Lower parabola is even-even.

							S	S26	S27	S28	S29	S30	S31	S32	S33	S34	S35	S36						
				16			115.21 444.89 1941		21 ms	125 ms	187 ms	1.178 s	2.572 s	0+	3/2+	0+	87.51 d	0+						
							^{+4,+6} 32.066 0.00168%	ECp,EC2p	ECp	ECp	EC	EC	EC	95.02	0.75	4.21	β	0.02						
								P	P24	P25	P26	P27	P28	P29	P30	P31	P32	P33	P34	P35				
				15			44.17 280.57 721		20 ms	260 ms	270.3 ms	4.140 s	2.498 m	1/2+	1+	100	14.292 d	25.34 d	12.43 s	47.3 s	1/2+			
							^{+3,+5} 30.973761 0.000549%	ECp	ECp	ECp	ECp,ECα	EC	EC	1/2+	1+	100	β	β	β	β	β			
								Si	Si22	Si23	Si24	Si25	Si26	Si27	Si28	Si29	Si30	Si31	Si32	Si33	Si34			
				14			14.03 3283		0 ms	103 ms	220 ms	2.234 s	4.16 s	0+	1/2+	0+	187.3 m	177 y	8.18 s	2.77 s	0+			
							^{+2,+4} 35.0855 0.003286%	ECp	ECp	ECp	ECp	EC	EC	92.23	4.67	3.10	β	β	β	β	β			
								Al	Al21	Al22	Al23	Al24	Al25	Al26	Al27	Al28	Al29	Al30	Al31	Al32	Al33			
				13			46.33 2719		70 ms	0.47 s	2.063 s	7.183 s	7.4E-5 y	5/2+	5+	100	β	β	β	β	β			
							⁺³ 26.981538 0.000277%	ECp	ECp	ECp	ECα	EC	EC	5/2+	5+	100	β	β	β	β	β			
								Mg	Mg20	Mg21	Mg22	Mg23	Mg24	Mg25	Mg26	Mg27	Mg28	Mg29	Mg30	Mg31	Mg32			
				12			650 1099		95 ms	127 ms	3.857 s	11.317 s	0+	5/2+	0+	9.488 m	20.91 h	1.30 s	335 ms	236 ms	120 ms	0+		
							⁺² 24.3050 0.003509%	ECp	ECp	EC	EC	EC	EC	78.99	10.00	11.01	β	β	β	β	β	β		
								Na	Na18	Na19	Na20	Na21	Na22	Na23	Na24	Na25	Na26	Na27	Na28	Na29	Na30	Na31		
				11			97.827 883		0 ms	447.9 ms	22.49 s	2.6019 y	3-	3/2+	4+	1.072 s	301 ms	30.5 ms	44.9 ms	48 ms	17.0 ms	3/2+		
							⁺¹ 22.989770 0.000187%	ECp	ECα	ECα	EC	EC	EC	100	β	β	β	β	β	β	β	β	β	
								Ne	Ne16	Ne17	Ne18	Ne19	Ne20	Ne21	Ne22	Ne23	Ne24	Ne25	Ne26	Ne27	Ne28	Ne29	Ne30	
							248.39 246.88 228.7		122 keV	109.2 ms	1/2-	1/2-	0+	3/2+	0+	37.24 s	3.38 m	602 ms	197 ms	32 ms	17 ms	0.2 s	0-	
							⁰ 20.1797 0.0112%	ECp,ECα	EC	EC	EC	EC	EC	90.48	0.27	9.25	β	β	β	β	β	β	β	
								F	F14	F15	F16	F17	F18	F19	F20	F21	F22	F23	F24	F25	F26	F27	F28	F29
							2012		2-	1.0 MeV	40 keV	64.49 s	109.77 m	1/2+	2-	4.158 s	4.23 s	2.23 s	0.34 s	59 ms				
							⁷ 1217.9		2p	ECp,ECα	EC	EC	EC	100	β	β	β	β	β	β	β	β	β	β
								O	O13	O14	O15	O16	O17	O18	O19	O20	O21	O22	O23	O24	O25	O26		
							7		8.58 ms	70.606 s	122.24 s	0+	5/2+	0+	26.91 s	13.51 s	3.42 s	2.25 s	82 ms	61 ms				
							⁸		3/2-	0-	1/2-	0+	5/2+	0+	5/2+	2-	4+	2.25 s	0+	0+				
							10		ECp	EC	EC	EC	EC	EC	β	β	β	β	β	β	β	β		
							20		99.762	0.838	0.200	β	β	β	β	β	β	β	β	β	β			

- Decay Q-value Range
- Q(??)
 - Q(β-)>0
 - Q(β-)-S_N>0
 - Q(β-)>0 + Q(EC)>0
 - Stable to Beta Decay
 - Q(EC)>0
 - Q(EC)-S_p>0
 - Q(P)>0
 - Naturally Abundant

Accelerators

INTRODUCTION: Uses of Accelerators

World wide inventory of accelerators, in total 15,000. The data have been collected by W. Scarf and W. Wieszczycka (See U. Amaldi Europhysics News, June 31, 2000)	
Category	Number
Ion implanters and surface modifications	7,000
Accelerators in industry	1,500
Accelerators in non-nuclear research	1,000
Radiotherapy	5,000
Medical isotopes production	200
Hadron therapy	20
Synchrotron radiation sources	70
Nuclear and particle physics research	110

Good Overview of accelerators (no equations) :

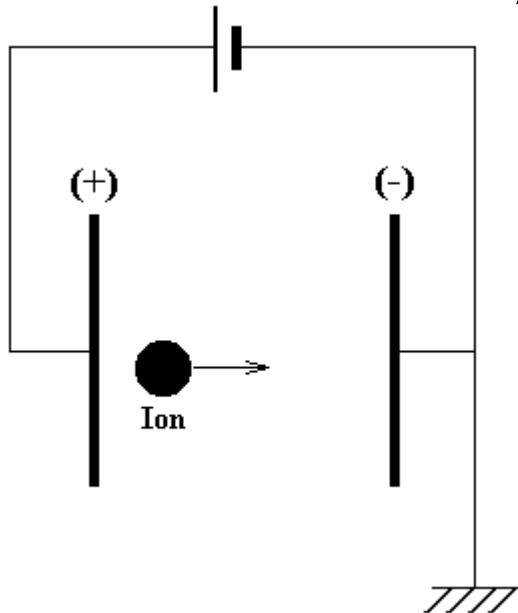
<http://nobelprize.org/physics/articles/kullander/>

I. Electrostatic Devices (constant E field)

Van de Graaf/Cockroft-Walton Accelerators
High Voltage Devices

A. Principle of Operation: One or Two Big Kicks

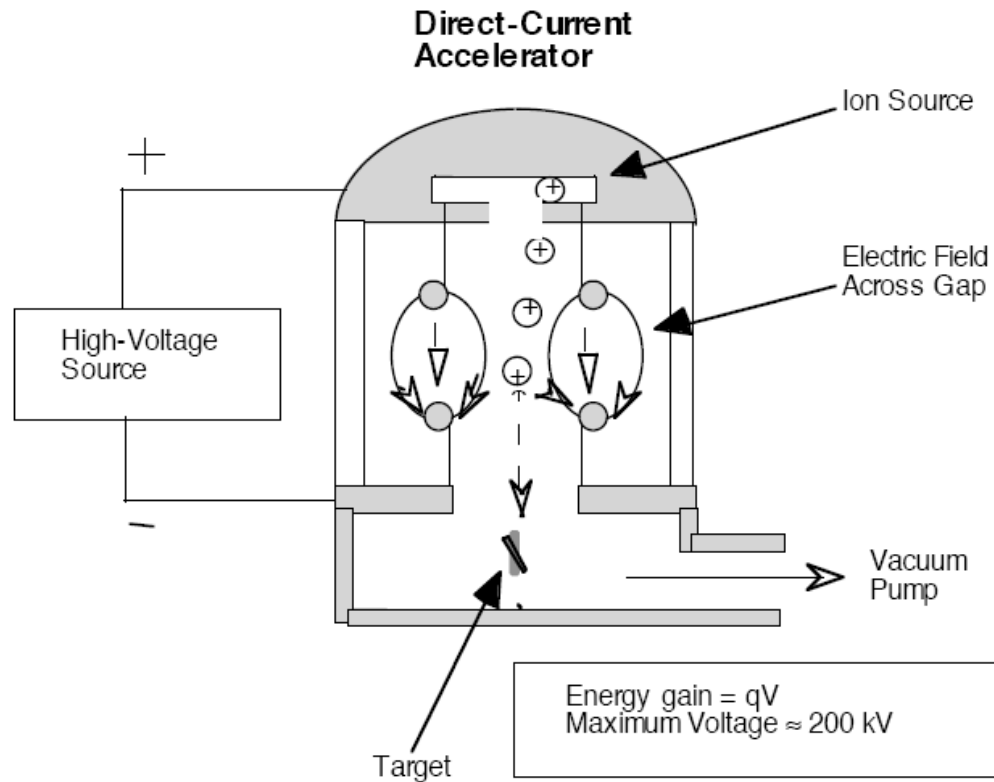
1. $\Delta E = qe\Delta V$ ΔE Change in KE of particle
 q Atomic charge state (ion charge)
 e Electric charge in units of eV
 ΔV Potential difference in volts



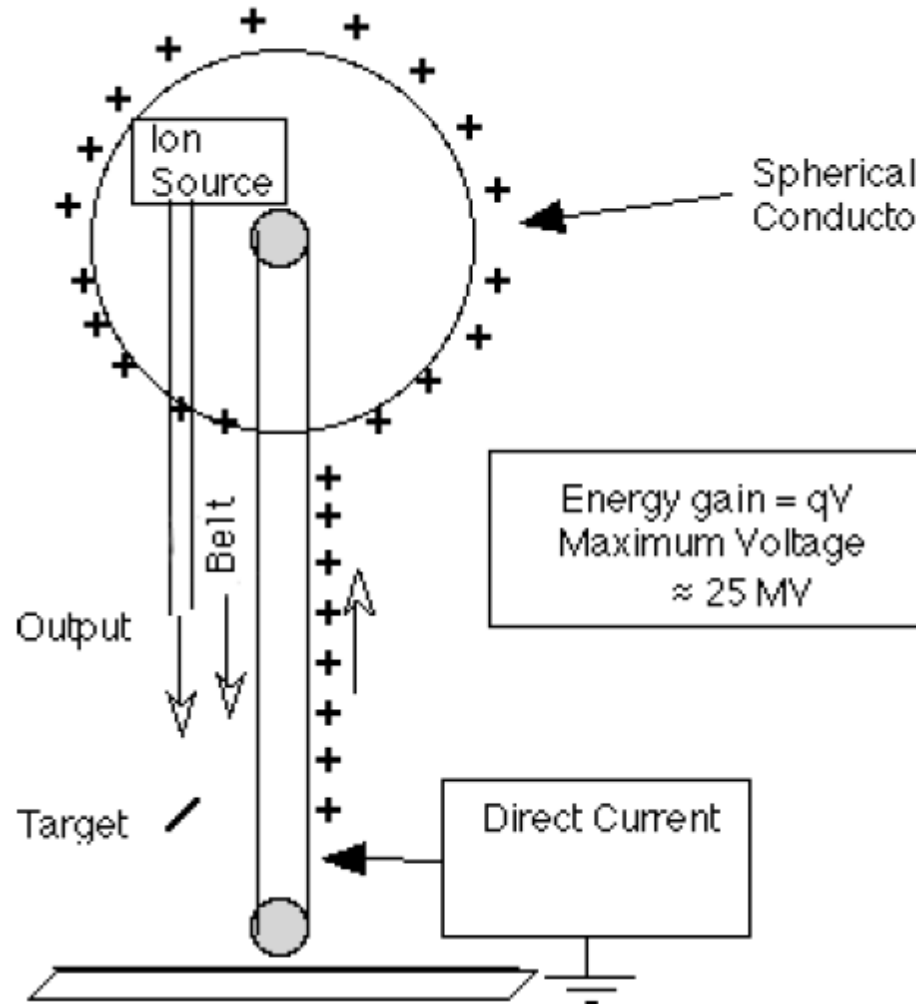
2. Limitations on ΔV

Electric discharge: $\Delta V \approx 25 \times 10^6$ Volts (Oak Ridge)

$\Delta E \approx (qe) 25$ MeV; SF_6 as insulator

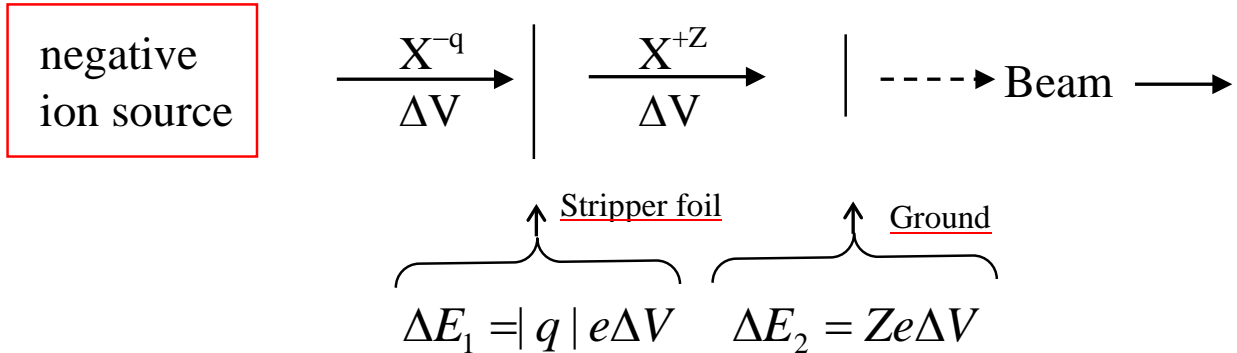


Van de Graaff Accelerator



Schematic of a Van de Graaf. Typically a voltage of 200 kV can be reached.
Problems: belt moves at $\sim 60 \text{ km/hr}$; Belt dust \rightarrow sparking; Need for an insulating gas (SF_6);

3. Tandem Van de Graaf: Two-step Acceleration



a. Total Energy Gain: $\Delta E = \Delta E_1 + \Delta E_2 = (|q| + Z) e \Delta V$

b. Example: S^{-2} ion ; terminal voltage = 25 MV
 $\Delta E = \{ |-2| + 16 \} 25 \text{ eMV} = 450 \text{ MeV}$

c. Large ΔV leads to higher charge state in second stage.

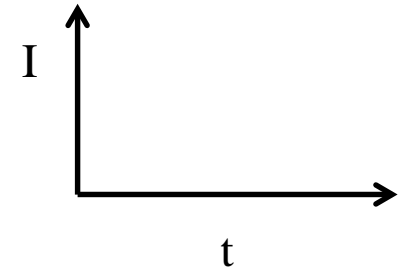


Tandem accelerator at Brookhaven National Lab. (BNL)

B. Properties

1. Ions: most of periodic table
2. $\Delta V \lesssim 25 \text{ MV}$; high precision, simple operation
3. $I \sim 10 \mu\text{A}$
4. Time structure: Continuous
5. Uses

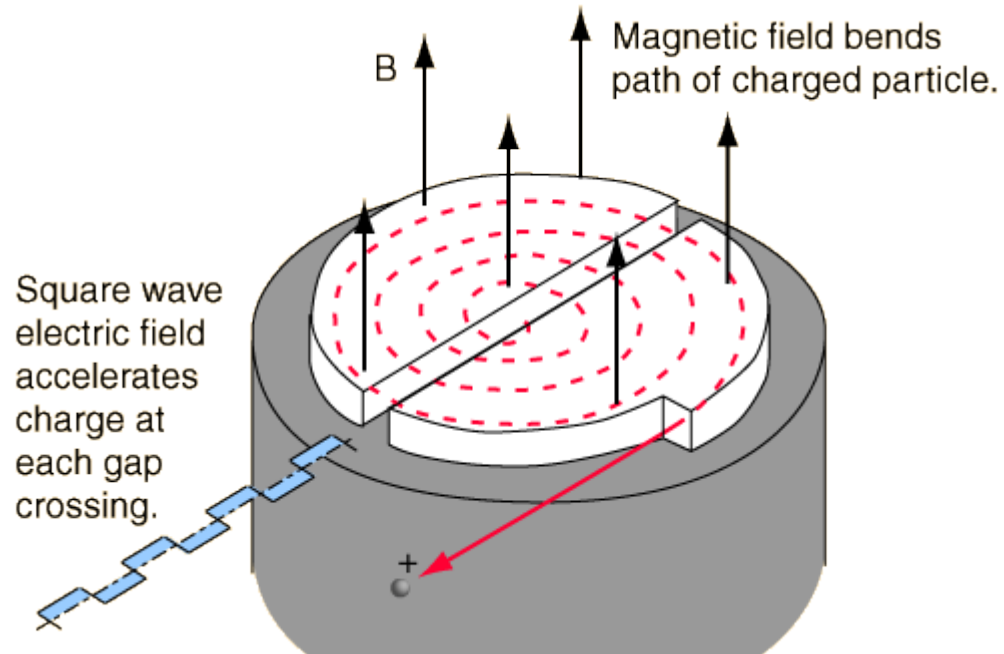
Largely applications today; e.g., ion implantation; charged-particle activation analysis ; ^{14}C dating.



II. Electrodynamics (Time varying E and B fields)

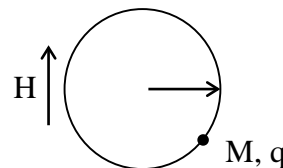
A. Cyclotron (Lawrence, 1929, Nobel Prize)

Idea: Confine the motion of the particle with a magnetic field while you accelerate it.



1. Equations of Motion for a Charged Particle in a Magnetic Field

Particle mass: M
 Charge state: qe
 Magnetic field: H
 radius: r



a. Trajectory is Circular path of radius r

$$F_{centripetal} = \frac{Mv^2}{r} \qquad F_{magnetic} = \frac{Hvqe}{c}$$

The two forces are balanced so equate them!

$$r = \frac{Mv^2 c}{Hvqe} = \left(\frac{Mc}{Hqe} \right) v \qquad \text{i.e. } r = f(v) \text{ (classically)}$$

b. Orbit time: $\underline{v \ll c}$

$$t = \frac{2\pi r}{v} = \frac{2\pi}{v} \frac{Mc v}{Hqe} = \frac{2\pi Mc}{Hqe} \qquad \text{CONSTANT!}$$

CYCLOTRON PRINCIPLE:

orbit time is independent
of particle energy for classical motion

ion-cyclotron
 resonance

c. Frequency- ω

$$\omega = \frac{2\pi}{t} = \frac{Hqe}{Mc} = \left(\frac{He}{c} \right) \frac{q}{M}$$

for $q/M \sim 0.5$ (e.g., ${}^4\text{He}^{+2}$, ${}^{12}\text{C}^{+6}$),
 $\omega \sim 10\text{-}30$ MHz for $H \sim 1.5$ tesla.
(lower end of FM frequency.)

Notice that for a fixed magnetic field H , the cyclotron frequency is proportional to q/M of the particle.

2. Acceleration

a. Supply radiofrequency energy for each revolution
i.e., $\Delta E = qe \Delta V$, where $\Delta V \sim 50\text{-}250$ kV

b. Result: velocity increases and particle spirals outward

c. Energy is limited by magnetic field H and radius r (\$)

d. Total energy: defined by number of orbits required to reach maximum radius, $r_{\text{max}} = n$: $\therefore \Delta E = n (qe) \Delta V$

e.g. for $n = 500$, $\Delta V = 200$ kV ($q = 2$), $\Delta E = 200$ MeV

3.

Classical Kinetic Energy: E_K

$$E_K = \frac{1}{2} M v^2 = \frac{1}{2} M \left(\frac{r^2 H^2 q^2 e^2}{M^2 c^2} \right) = \frac{1}{2} \underbrace{\left(\frac{r^2 H^2 e^2}{c^2} \right)}_{\text{cyclotron}} \underbrace{\left(\frac{q^2}{A} \right)}_{\text{ion}}$$

$$E_K = K \frac{q^2}{A}$$

For $v < c$; limited by relativity

K is the figure of merit for the cyclotron

If we insert values for the constants we get:

$$E_K = 5.05 \times 10^{-3} \text{ H}^2 r^2 (q^2/A)$$

$$\text{MeV/tesla}^2\text{-cm}^2$$

B. Properties

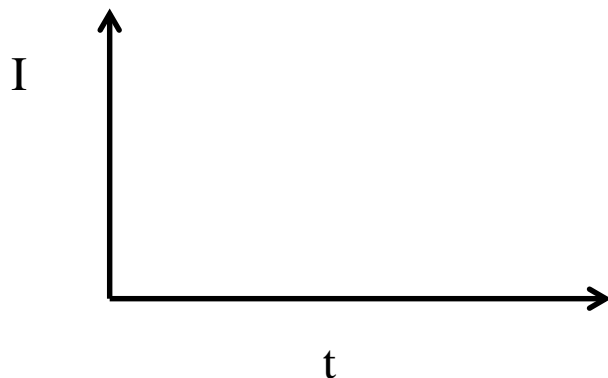
1. Ions: Most of periodic table (electron cyclotron resonance (ECR) sources yield high q) ion sources permit up to U ions

2. Higher energy, less precision than Van de Graafs

3. Energy limits: H and He: $K = 215$ (IU); $K=500$ (TRIUMF/CANADA)
Heavy ions: $K = 1200$ (MSU)

4. Intensity: $I \lesssim 10\mu\text{A}$

5. Time structure of beam: Pulses



More historical information:

<http://www.aip.org/history/lawrence/>

Original paper on cyclotrons:

http://prola.aps.org/abstract/PR/v40/i1/p19_1

Facts about the IU cyclotron (IUCF) :

<http://www.iucf.indiana.edu/whatis/facts.php>