What did you learn in the last lecture?

What did you learn in the last lecture?

Beta stability, the LD Mass Formula, and Accelerators

Simplest form of LD Mass Formula

$$TBE = C_1 A - C_2 A^{2/3} - C_3 Z^2/A^{1/3} - C_4 (N\text{-}Z)^2/A^2 + C_6 \delta/A^{1/2}$$

$$\langle BE \rangle = C_1 - C_2 A^{-1/3} - C_3 Z^2/A^{4/3} - C_4 (N\text{-}Z)^2/A^3 + C_6 \delta/A^{3/2}$$

Line of Beta Stability – Isobars

1. Beta Decay – Form of Radioactive Decay

$n \leftrightarrow p$ conversion inside nucleus

$\therefore$ **A** doesn't change; just N/Z ratio – ISO BARS

Most probable N/Z ratio $\equiv$ Line of Beta Stability

2. Example: A = 75 chain

N/Z= 1.5 N/Z= 1.08

$$^{75}_{30}Zn \rightarrow {}^{75}_{31}Ga \rightarrow {}^{75}_{32}Ge \rightarrow {}^{75}_{33}As \leftarrow {}^{75}_{34}Se \leftarrow {}^{75}_{35}Br \leftarrow {}^{75}_{36}Kr$$

$n \rightarrow p$ ↑ $n \leftarrow p$

Beta-
Stable
Nucleus

3. Isobaric Mass Formula

Since in β decay the mass number remains constant, it is useful to develop an isobaric mass formula ("iso" means same in Greek).

- A is constant in Binding Energy Equation (& <BE> form)

$$\left(\Delta {}^{A}_{Z}X \right) = Z\,\Delta_H + N\,\Delta_n - TBE$$

plug in LD Mass Equation

- $\left(\Delta {}^{A}_{Z}X \right) = d_1\, Z^2 + d_2 Z\, d_3 + d + d_4\delta$, where $d_i = f\,(Ci, A)$

- What is the functional form of this equation? minimum defines most stable nuclide for a given A. These values define the "valley of stability".

Atomic number of this nucleus is Z_A.

Valley (line) of Beta stability

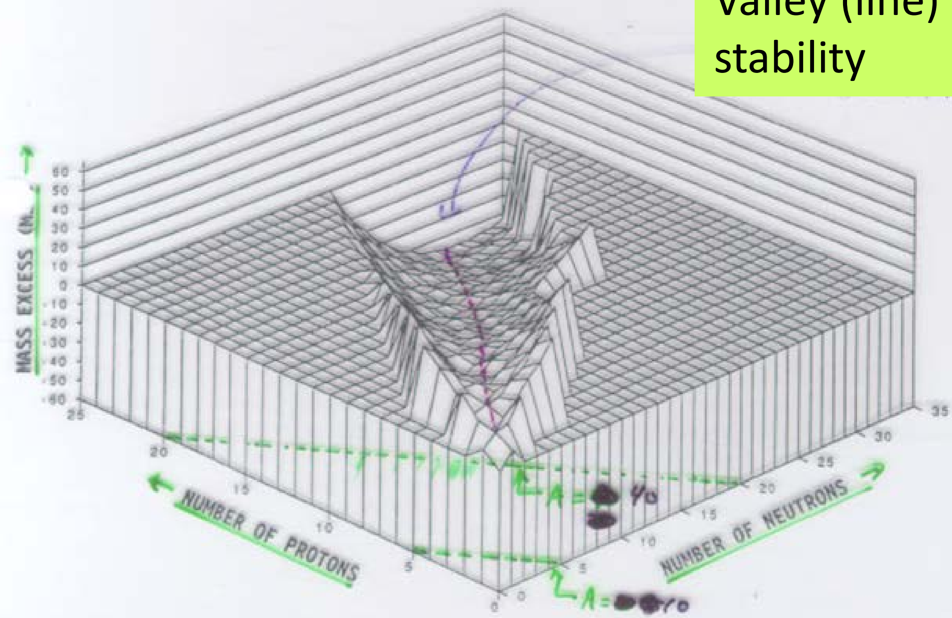


Figure 3.8. A three-dimensional representation of the valley of beta stability

Two Cases

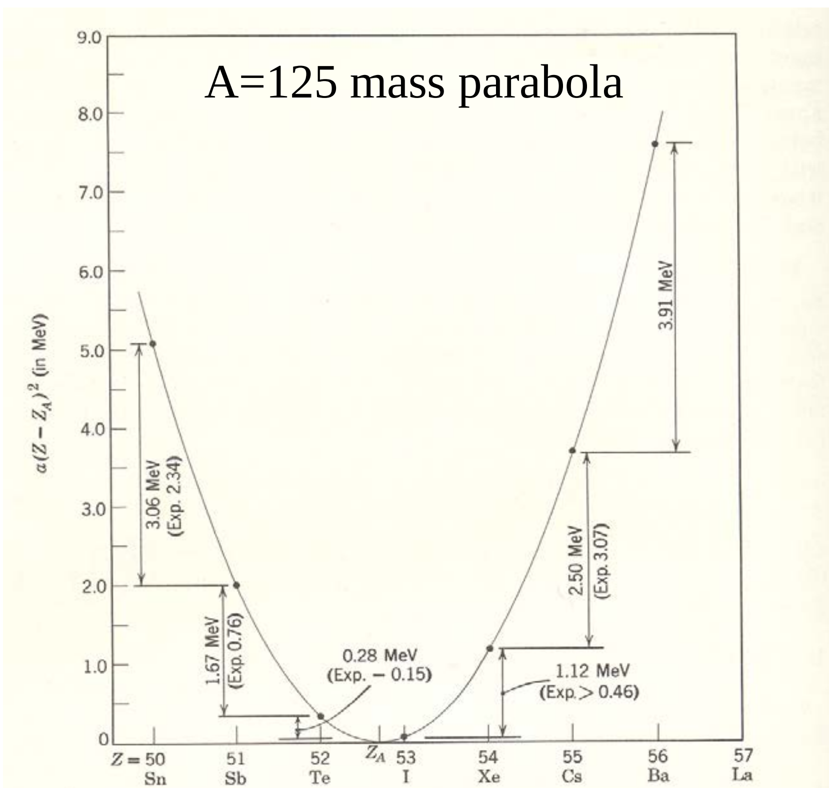
a. Case I: odd-A nuclei ($\delta = 0$)

- Single parabola: $\Delta\left({}^{A}_{Z}X\right) = d_1 Z^2 + d_2 Z + d_3$; one Parabola

- Most probable charge: $\dfrac{\partial \Delta({}^{A}_{Z}X)}{\partial Z} = 2d_1 Z_A + d_2$

RESULT: **ONE STABLE ISOTOPE PER MASS NUMBER**

M(Z)-M(Z_A) in MeV



Z

Consider the two mass parabolas of A=75 and A=157. What do you notice?

$M(Z) - M(Z_A)$

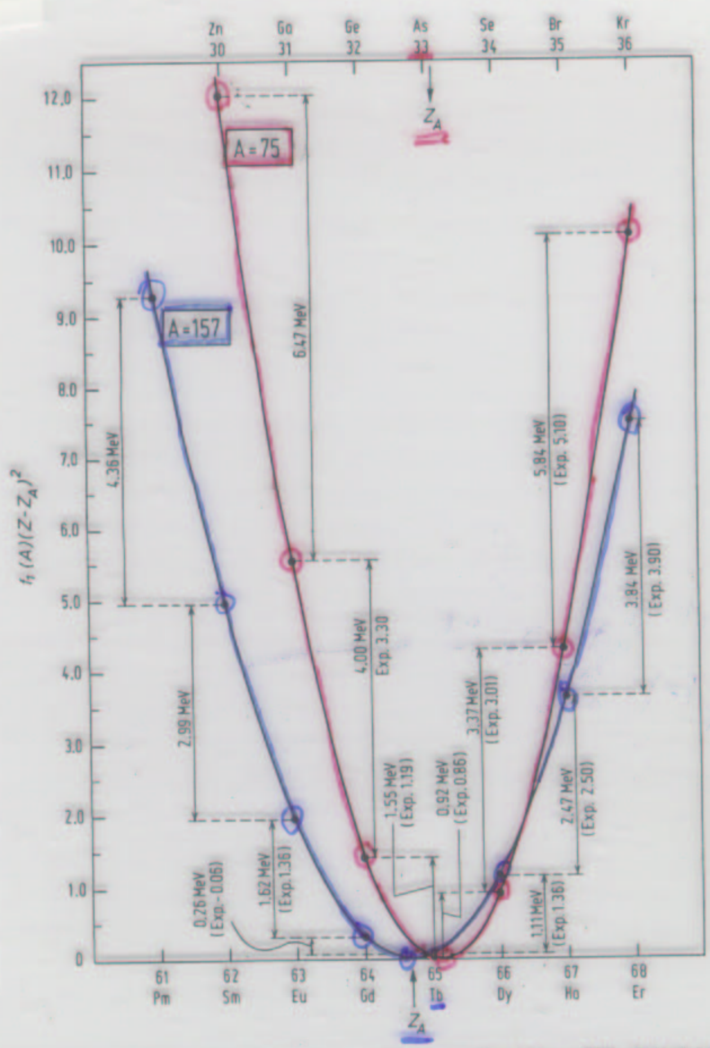
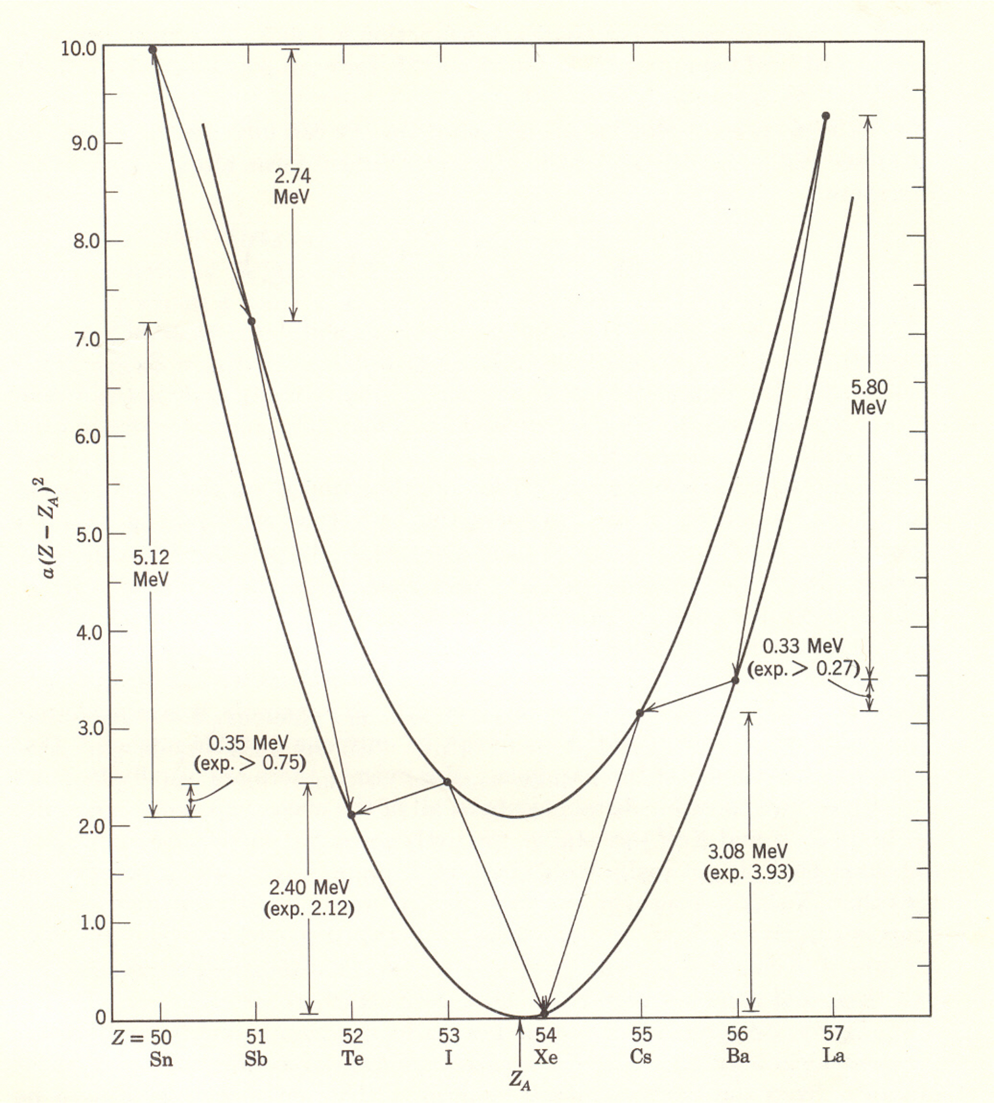


Fig. 2-7 Mass parabolas for $A = 75$ and $A = 157$, as calculated from (2-8). Calculated mass differences between neighboring isobars are indicated, with experimentally determined values shown in parentheses for comparison. The top Z scale refers to $A = 75$, the bottom one to $A = 157$.

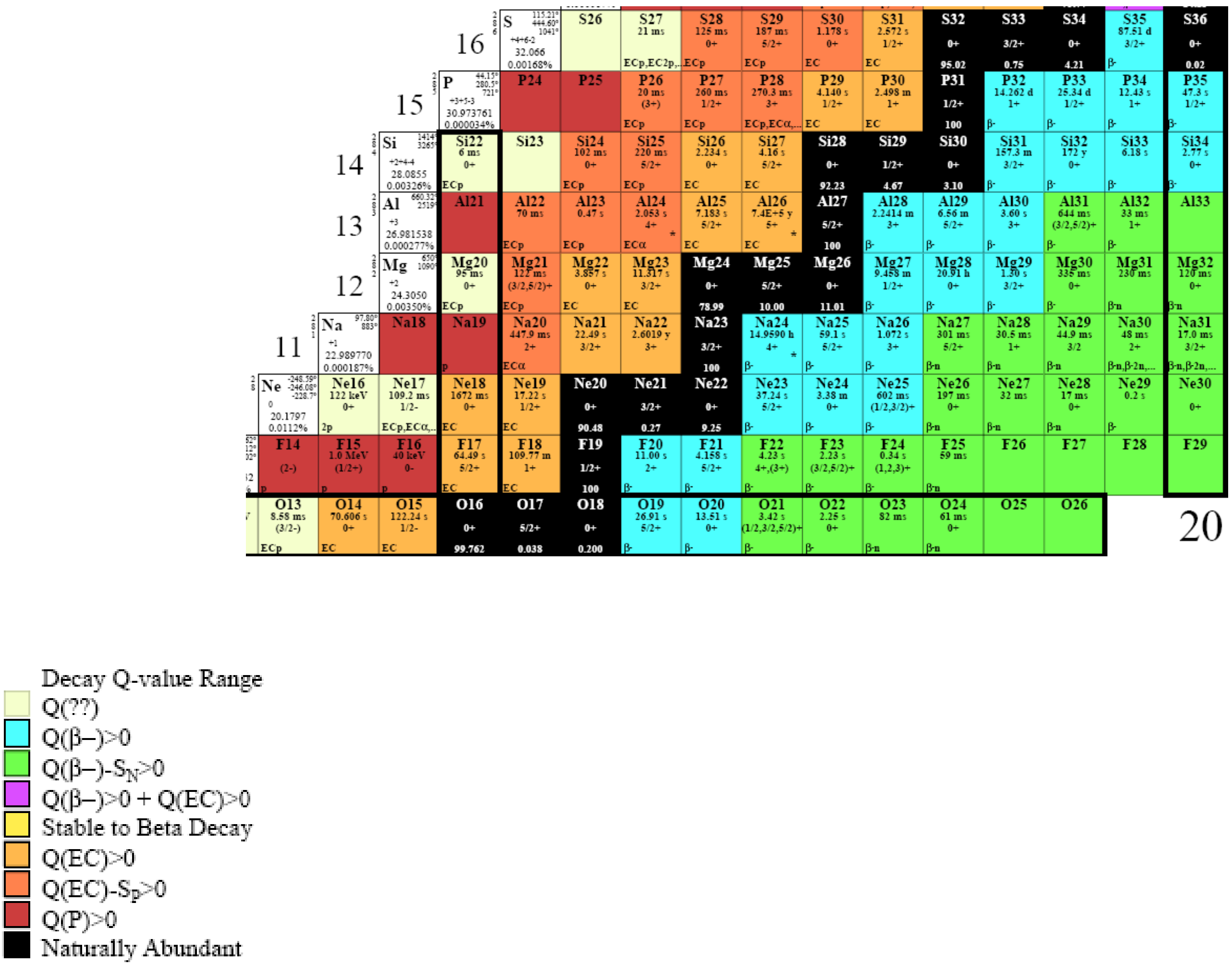
b. Case II: even A nuclei $(\delta = \pm 1)$

$$(\Delta {}^{A}_{2}X) = d_1 Z^2 + d_2 Z + d_3 \pm \delta\, d_4$$

RESULT: Two parabolae ; even-Z always lower CAN HAVE 1,2, OR 3 STABLE NUCLEI PER A.



- A=128
- Upper parabola is odd-odd
- Lower parabola is even-even.


16
15
14
13
12
11
20
Decay Q-value Range
Q(??)
Q(β–)>0
Q(β–)-S_N>0
Q(β–)>0 + Q(EC)>0
Stable to Beta Decay
Q(EC)>0
Q(EC)-S_P>0
Q(P)>0
Naturally Abundant

Accelerators

INTRODUCTION: Uses of Accelerators

World wide inventory of accelerators, in total 15,000. The data have been collected by W. Scarf and W. Wiesczycka (See U. Amaldi Europhysics News, June 31, 2000)	
Category	**Number**
Ion implanters and surface modifications	7,000
Accelerators in industry	1,500
Accelerators in non-nuclear research	1,000
Radiotherapy	5,000
Medical isotopes production	200
Hadron therapy	20
Synchrotron radiation sources	70
Nuclear and particle physics research	110

Good Overview of accelerators (no equations) :

http://nobelprize.org/physics/articles/kullander/

I. Electrostatic Devices (constant E field)

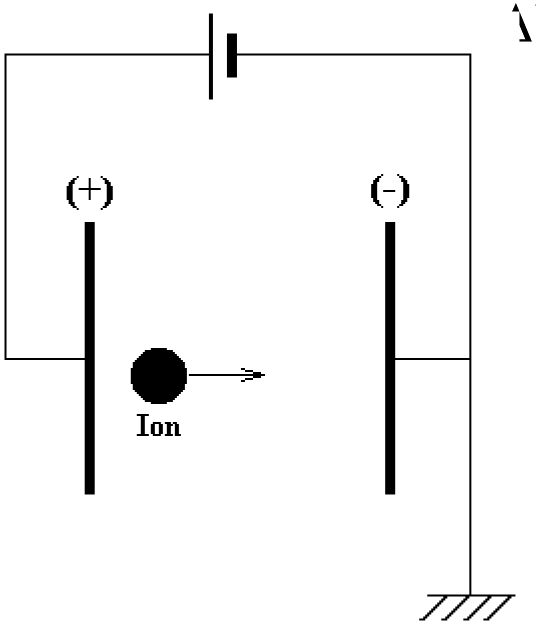
Van de Graaf/Cockroft-Walton Accelerators

High Voltage Devices

A. Principle of Operation: One or Two Big Kicks

1. $\Delta E = qe\Delta V$

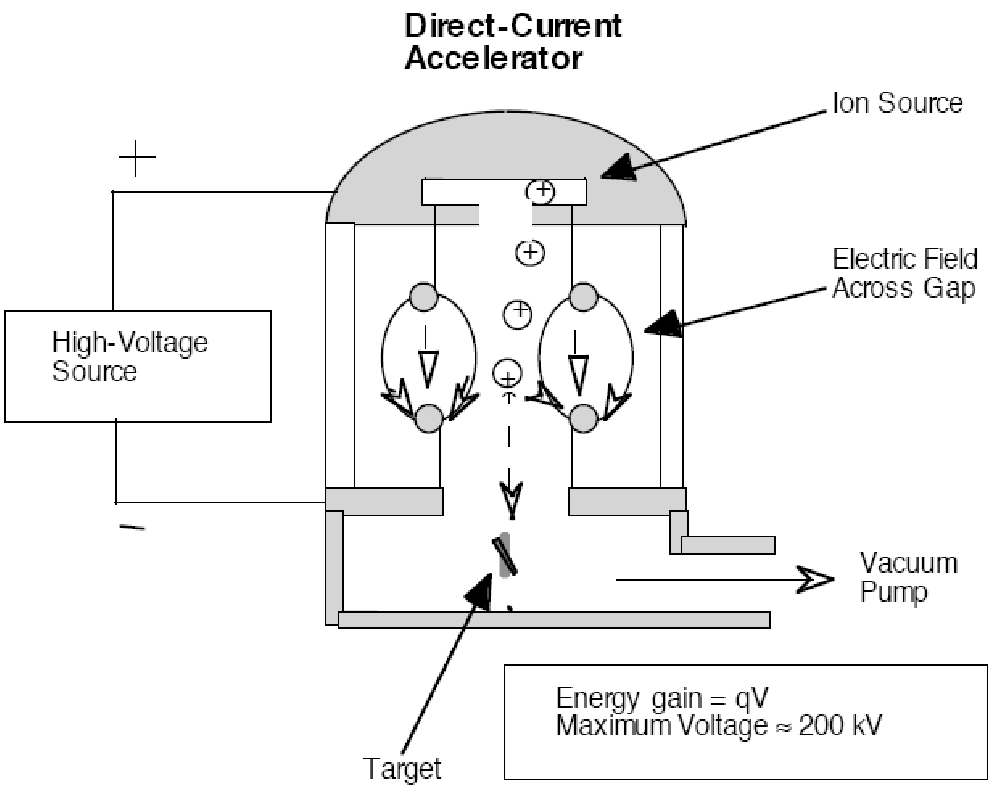
ΔE	Change in KE of particle
q	Atomic charge state (ion charge)
e	Electric charge in units of eV
ΔV	Potential difference in volts

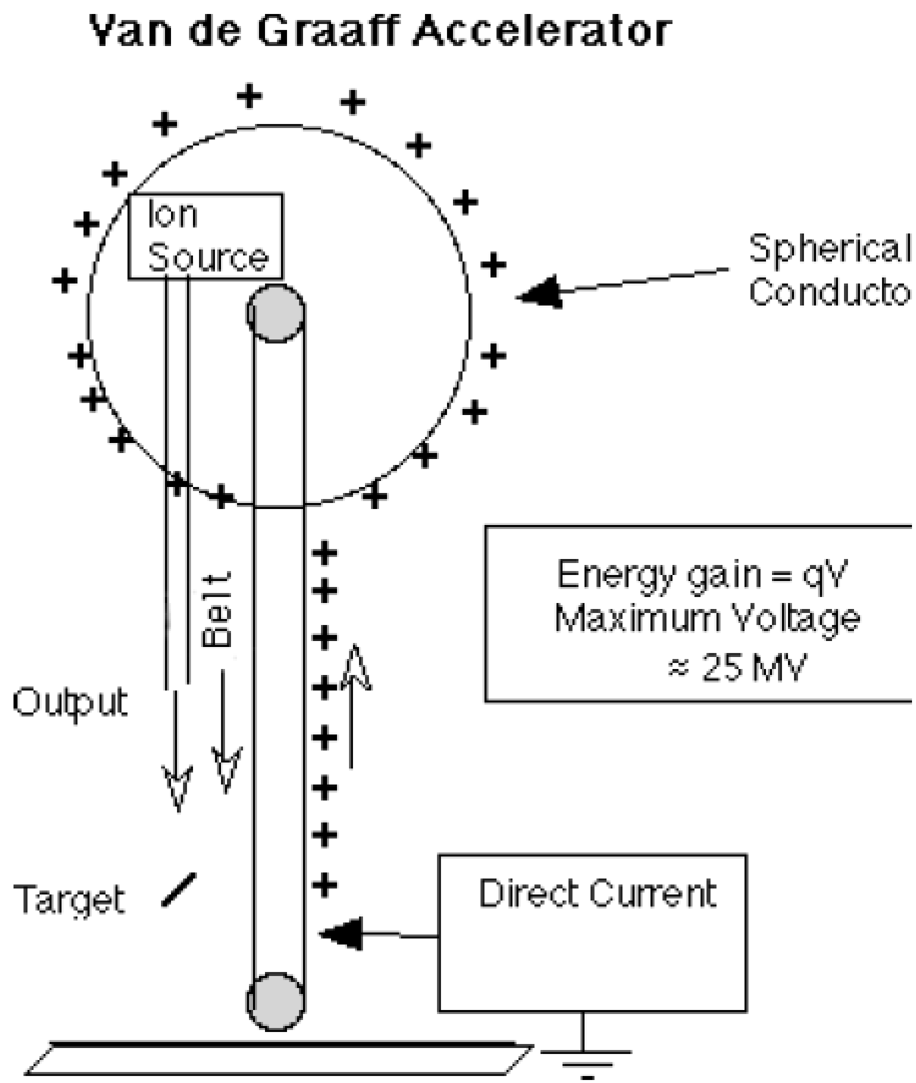


2. Limitations on ΔV

Electric discharge: $\Delta V \approx 25 \times 10^6$ Volts (Oak Ridge)

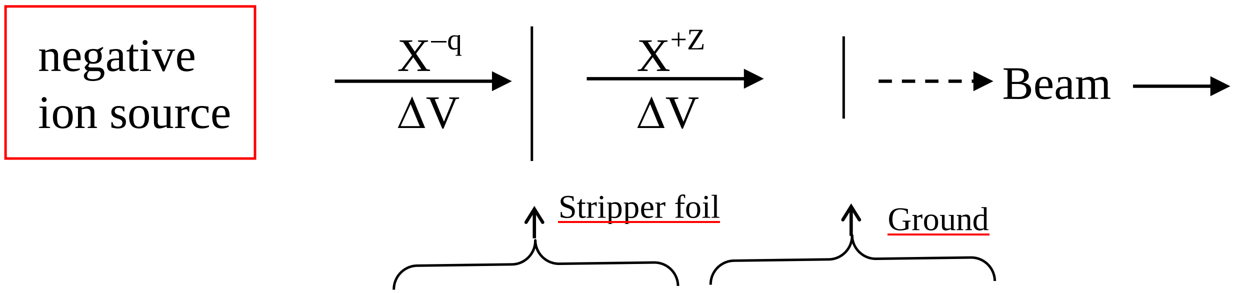
$\Delta E = \approx$ (qe) 25 MeV; SF_6 as insulator





Schematic of a Van de Graaf. Typically a voltage of 200 kV can be reached.
Problems: belt moves at ~ 60 km/hr; Belt dust ➔ sparking; Need for an insulating gas (SF_6);

3. Tandem Van de Graaf: Two-step Acceleration



$$\Delta E_1 = |q|\, e\Delta V \qquad \Delta E_2 = Ze\Delta V$$

a. Total Energy Gain: $\Delta E = \Delta E_1 + \Delta E_2 = (|q| + Z)\, e\, \Delta V$

b. Example: S^{-2} ion ; terminal voltage = 25 MV

$\Delta E = \{ |-2| + 16 \}\ 25\ \text{eMV} = 450\ \text{MeV}$

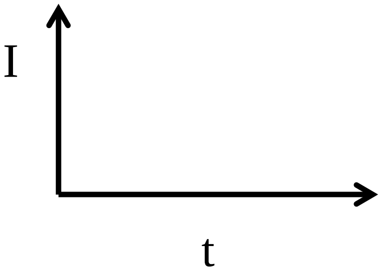
c. Large ΔV leads to higher charge state in second stage.

Tandem accelerator at Brookhaven National Lab. (BNL)

B. Properties

1. Ions: most of periodic table
2. $\Delta V \lesssim 25$ MV ; high precision, simple operation
3. I ~ 10μA
4. Time structure: Continuous
5. Uses

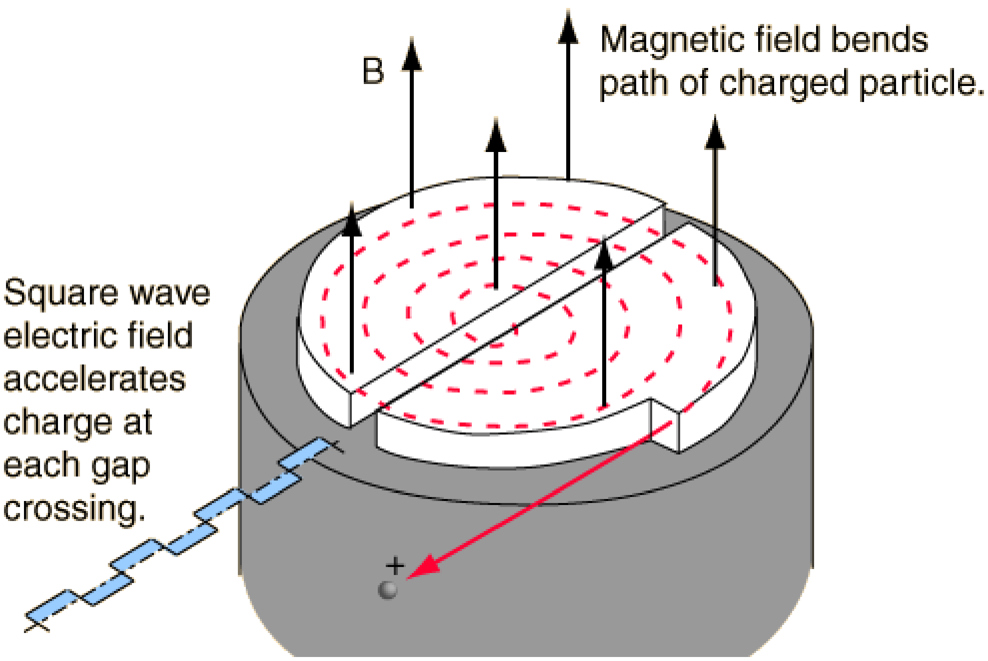
   Largely applications today; e.g., ion implantation; charged-particle activation analysis ; ^{14}C dating.



II. Electrodynamic (Time varying E and B fields)

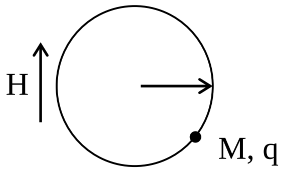
A. Cyclotron (Lawrence, 1929, Nobel Prize)

Idea: Confine the motion of the particle with a magnetic field while you accelerate it.



1. Equations of Motion for a Charged Particle in a Magnetic_Field

Particle mass: M
Charge state: qe
Magnetic field: H
radius: r



a. Trajectory is Circular path of radius r

$$F_{centripetal} = \frac{Mv^2}{r} \qquad F_{magnetic} = \frac{Hvqe}{c}$$

The two forces are balanced so equate them!

$$r = \frac{Mv^2c}{Hvqe} = \left(\frac{Mc}{Hqe}\right)v$$

i.e. r = f(v) (classically)

b. Orbit time: <u>v << c</u>

$$t = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{Mcv}{Hqe} = \frac{2\pi Mc}{HqE}$$

CONSTANT!

<u>CYCLOTRON PRINCIPLE:</u> <u>orbit time</u> is <u>independent</u> of <u>particle energy</u> for <u>classical motion</u>

ion-cyclotron resonance

c. Frequency-ω

$$\omega = \frac{2\pi}{t} = \frac{Hqe}{Mc} = \left(\frac{He}{c}\right)\frac{q}{M}$$

for q/M ~ 0.5 (e.g., $^{4}He^{+2}$, $^{12}C^{+6}$),
ω ~ 10-30 MHz for H ~ 1.5 tesla.
(lower end of FM frequency.)

Notice that for a fixed magnetic field H, the cyclotron frequency is proportional to q/M of the particle.

2. Acceleration

a. Supply radiofrequency energy for each revolution

i.e., $\Delta E = qe\,\Delta V$, where $\Delta V \sim$ 50-250 kV

b. Result: velocity increases and particle spirals outward

c. Energy is limited by magnetic field H and radius r ($)

d. Total energy: defined by number of orbits required to reach maximum radius, $r_{max} = n$: $\therefore\ \Delta E = n\,(qe)\,\Delta V$

e.g. for n = 500, ΔV = 200 kV (q = 2) , ΔE = 200 MeV

3. <u>Classical Kinetic Energy: E_K</u>

$$E_K = \frac{1}{2}Mv^2 = \frac{1}{2}M\left(\frac{r^2H^2q^2e^2}{M^2c^2}\right) = \underbrace{\frac{1}{2}\overbrace{\left(\frac{r^2H^2e^2}{c^2}\right)}^{\text{cyclotron}}}_{K}\overbrace{\left(\frac{q^2}{A}\right)}^{\text{ion}}$$

$$E_K = K\frac{q^2}{A}$$

For v < c; limited by relativity
K is the figure of merit for the cyclotron

If we insert values for the constants we get:

$E_K = 5.05 \times 10^{-3}\ H^2\ r^2\ (q^2/A)$ MeV/tesla^2-cm^2

B. Properties

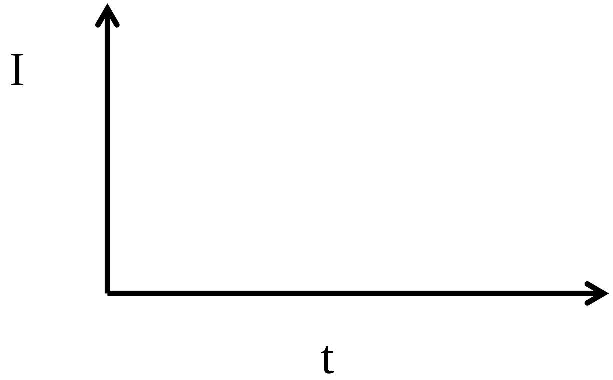
1. Ions: Most of periodic table (electron cyclotron resonance (ECR) sources yield high q) ion sources permit up to U ions

2. Higher energy, less precision than Van de Graafs

3. Energy limits: H and He: K = 215 (IU); K=500 (TRIUMF/CANADA)
Heavy ions: K = 1200 (MSU)

4. Intensity: $I \lesssim 10\mu A$

5. Time structure of beam: Pulses



More historical information:
http://www.aip.org/history/lawrence/
Original paper on cyclotrons:
http://prola.aps.org/abstract/PR/v40/i1/p19_1
Facts about the IU cyclotron (IUCF) :
http://www.iucf.indiana.edu/whatis/facts.php