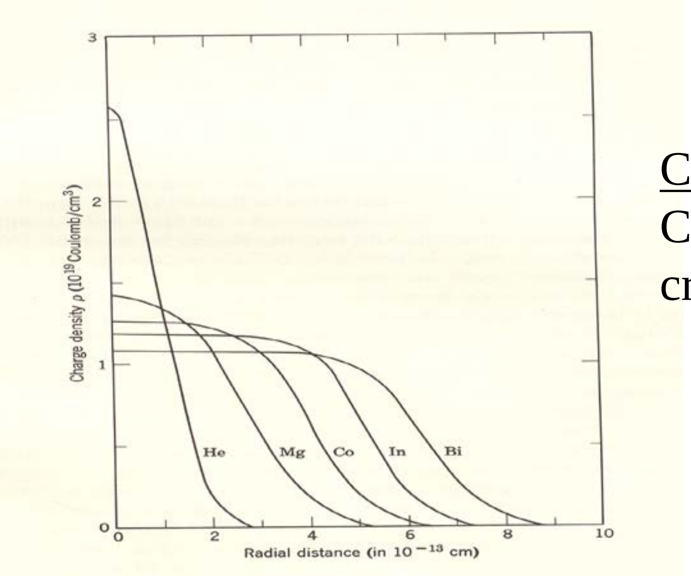
What did you learn in the last lecture?

Charge density distribution of a nucleus from electron scattering SLAC: 21 GeV e^-'s ; $\lambda \sim 0.1$ fm

(to first order assume that this is also the matter distribution of the nucleus)



CONCLUSION:
Central uniform density and diffuse surface (cloudy crystal ball)

Nucleus of A Nucleons

a. First Approximation: Uniform Density Sphere
Volume $V \cong A \times V_{nucleon} = (4/3)\,\pi R^3$; $V_{nucleon} \approx$ constant

b. Liquid Drop Analogy: Assume all nucleons uniformly distributed for V

then, $R = [(3A \bullet V_{nucleon})/4\pi\,]^{1/3} = r_0 A^{1/3}$; r_0 is the nuclear radius constant.
$r_0 \approx 1.2 - 1.4$ fm (ALWAYS GIVEN)

Calculate the radius of $^{216}_{84}\mathrm{Po}$ Given $r_0 = 1.40$ fm

$R = r_0 A^{1/3} = (1.40 \text{ fm})(216)^{1/3} = 8.40 \text{ fm} = 8.40 \times 10^{-13}$ cm

b. The density distribution of real nuclei is described by a Woods-Saxon Shape (Fermi function)

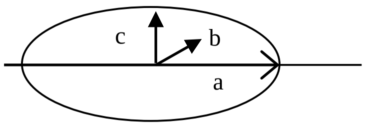
$$\rho(r) = \frac{\rho_0}{1 + e^{-(r - R_{1/2})/d}}$$

ρ_0 = central density (~ 2×10^{14} g/cm^3)
$R_{1/2}$ = half-density radius ; $R_{1/2} = r_0 A^{1/3}$, $r_0 = 1.07$ fm
i.e., radius at which $\rho = \rho_0/2$
d = diffuseness ; distance over which $\rho(r)$ decreases from 0.90 ρ_0 to 0.10 ρ d~ 2.4 fm

Nuclear Shapes

1. Spherical : Near N or Z = 2,8,20,28,50,82, and 126 (neutrons) ⟸ Magic numbers

2. Spheroidal : For nucleon numbers midway between magic number

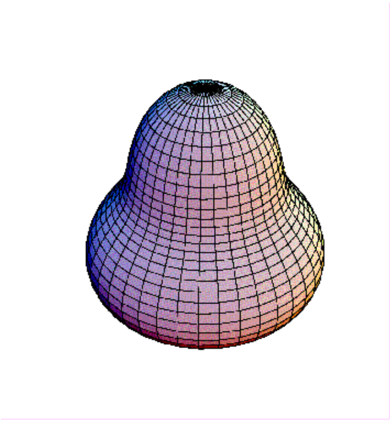


i) Prolate: $a > b = c \Rightarrow$ rugby ball

ii) Oblate: $a < b = c \Rightarrow$ discus

3. Exotic Shapes

Octupole (pear-shaped) ; fission (dumbell) ;



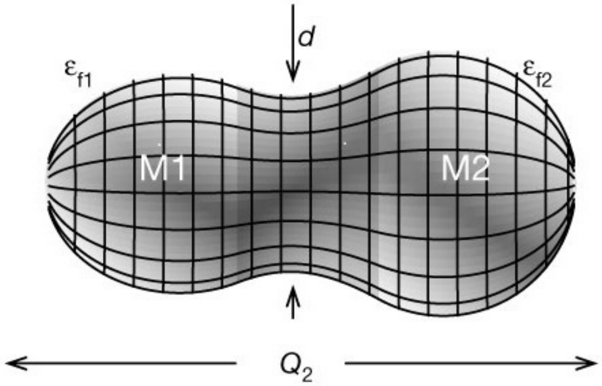


Fig 3.1
Ehmann

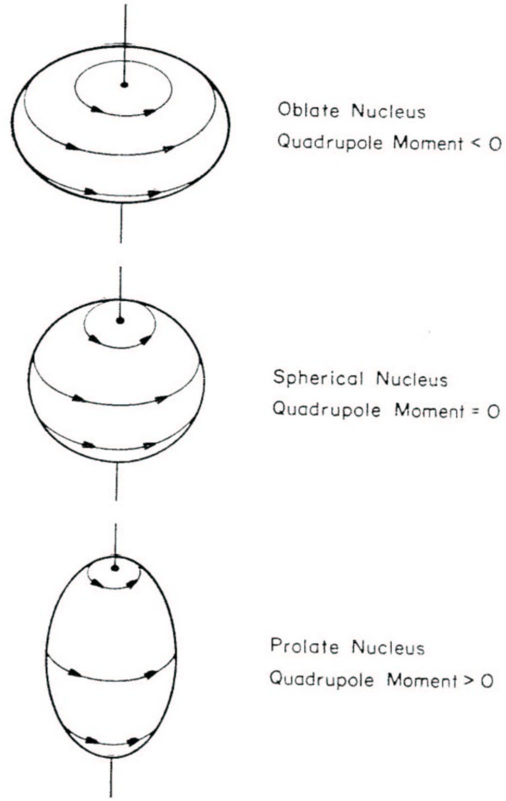


Figure 3.1. Shapes of the nucleus.

Motion of the charged particles (protons) within a nucleus represents a current just as the electrons moving through a wire do. Consequently a magnetic field is generated by the nucleus. This field is static (unchanging) and is called the magnetic moment of the nucleus. This magnetic moment tells us about the shape of the nucleus.

Nuclear Models

Philosophy and Difficulties

- Nuclear Force – no analytical expression
- Many-Body Problem – no mathematical solution

} $\Rightarrow$ computer approximations

MODELS

Macroscopic Properties
Energetics, Sizes, Shapes
Assumes all nucleons are alike
(except charge)

MicroscopicModels
Spins, Quantum States, Magic Numbers
Assumes all nucleons are different

Unified Model

Liquid Drop Model (Neils Bohr- 1940's)

Assumption: The nucleus is a charged, nonpolar liquid drop,
Chemical analogy: a cluster of Xe and Xe^+ atoms held together by Van der Waals attractions

Justification

1. Nuclear Behavior: Similarities to liquid drop

a. Force is short-ranged ; i.e., "sharp" boundary at surface

b. Force is saturated; i.e. all nucleons in bulk of the liquid are bound equally, independent of radius

c. Nucleus is incompressible at low temperatures – accounts for uniform density and constant <BE>

d. Surface Tension – Surface nucleons lose binding ; ∴ spherical

2. Differences

a. Few Particles ; $A \lesssim 270$ vs 10^{23}

b. Protons carry charge

c. Two types of particles

d. Result: microscopic properties exert significant influence and modify simple results.

Contributions to the Total Binding Energy of a Charged Liquid Drop

1. Attractive Forces

STRONG NUCLEAR FORCE: EQUAL AMONG ALL NUCLEONS

Rationale: <BE> curve (Fig. 2.1)

<BE> ~ 8 MeV = TBE/A

$\therefore$ TBE = C_1 A = E_v , where E_v is called the volume term

NOTE: TBE α Volume = 4/3 π R^3

and C_1 is a constant related to the strength of nuclear force

THIS IS THE PRIMARY ATTRACTIVE TERM (~ only term in neutron stars)

2. Loss of Binding Energy

Nucleus is a small system and also contains charged particles; two components (n & p)

Major **loss** terms: subtract from attractive term

a. Surface Tension: Surface Energy -- E_s

Nucleons on surface lose binding relative to bulk

Surface Energy E_s : DECREASES TOTAL BINDING ENERGY

$E_s = -4\pi\sigma R^2$, where σ = surface-tension constant

$E_s = -4\pi\sigma(r_0 A^{1/3})^2$ and $4\pi R^2$ is the surface area of a sphere

$E_s = -C_2 A^{2/3}$ ∴ small nuclei lose binding to greater extent than large nuclei ($A^{2/3}/A$)

b. Electric Charge: **Coulomb Energy -- E_c**

Protons are charged and repel one another – <u>DECREASES BINDING</u>

Coulomb's Law: $$E_C = -\frac{3}{5}\frac{Z^2 e^2}{R}$$ Electrostatic energy of a charged sphere

$$E_c = C_3\, Z^2/A^{1/3}$$

∴ Nuclei with large atomic numbers lose binding (Z^2)

⇒ E_s, and E_c ARE MAJOR SOURCES OF BE LOSS

c. Two-Component Liquid: Symmetry Energy – E_{sym}

$$E_{sym} = -C_4 \frac{(N-Z)^2}{A^2}$$

i.e., minimum E loss when N = Z

d. Diffuse Surface Correction

$E_{diff} = -C_5 Z^2/A$; small ; we'll ignore

e. Pairing Effect

- Small but systematic differences depending on even-odd character of nucleus
- Nucleons of the same type prefer to exist in pairs (n-n, p-p) (anti-Hund's rule)
- Empirical evidence: stable nuclei

even Z, even N	even Z, odd N	odd Z, even N	odd Z, odd N
(e-e)	(e-o)	(o-e)	(o-o)
157	55	50	4

- Pairing energy

$$E_p = C_6\, \delta / A^{1/2} \quad \text{where } \delta = \begin{cases} +1 & \text{e-e} \\ 0 & \text{e-o \& o-e} \\ -1 & \text{o-o} \end{cases}$$

i.e., extra binding for e-e and extra loss for o-o

Additional Terms

C_1 to C_6 give fundamental information about nuclear matter; Equations with up to 250 parameters have been added (based on ~2500 pieces of data). However, fit improvement is small and physical significance of C_i's is modified.

Semi empirical Mass Equation: Nuclear Equation of State (T=0)

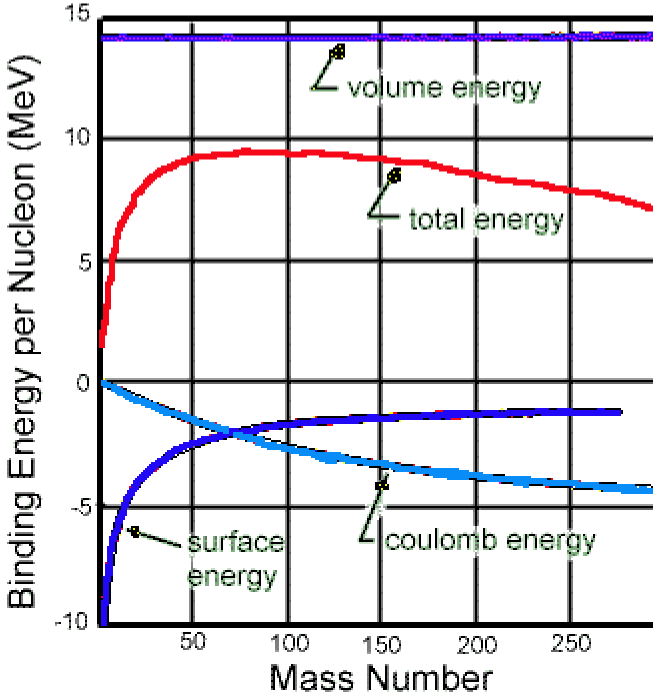
(half theory/half data)

Equation summarizes the macroscopic properties of nuclei in their ground states (T=0).
Useful for predictive purposes, searching for new isotopes and elements, and astrophysics

$$\mathrm{TBE} = C_1 A - C_2 A^{2/3} - C_3 Z^2/A^{1/3} - C_4 (N-Z)^2/A^2 + C_6 \delta/A^{1/2}$$

$$\langle \mathrm{BE} \rangle = C_1 - C_2 A^{-1/3} - C_3 Z^2/A^{4/3} - C_4 (N-Z)^2/A^3 + C_6 \delta/A^{3/2}$$

C_1	$C_2 A^{-1/3}$	$C_3 Z^2/A^{4/3}$	$C_4 (N-Z)^2/A^3$	$C_6 \delta/A^{3/2}$
Nuclear Attraction	Surface Loss	Charge Loss	Asymmetry Loss	Pairing Gain/Loss



Constants C_i

a. Derive from ~ 2500 known nuclear masses
with only C_1–C_6 terms get ± 5.0 MeV accuracy (0.005u)

b. Deviations: Evidence for shell structure and sharp changes

Nuclear Shell Model
proposed independently
by Maria Goeppert Mayer
and Johannes H. D. Jenson
in 1949.

(Nobel Prize in
Nuclear Physics to
Goeppert Mayer in 1963).

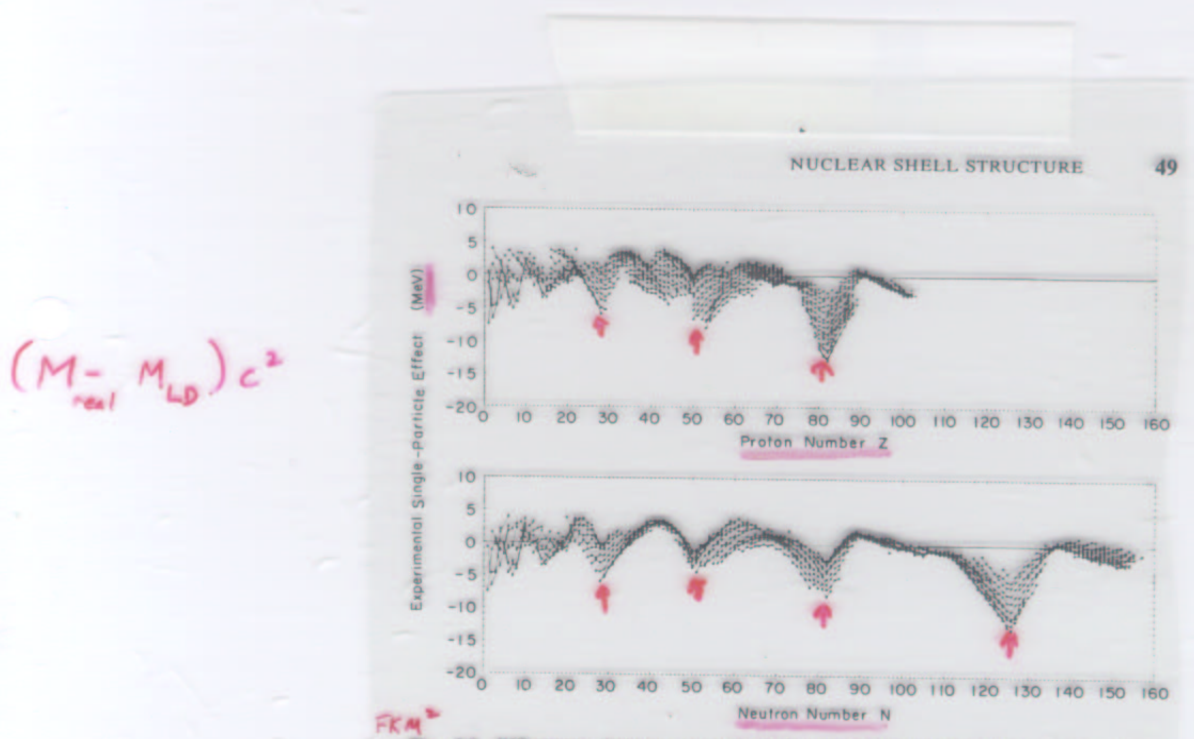


Fig. 2-9 Differences between experimental and liquid-drop-formula masses. In top graph isotones, in bottom graph isotopes are connected by lines. (From reference M2; drawing made available by J. R. Nix.)

c. Physical significance

C_1	$\Rightarrow$	strength of nuclear force
C_2	$\Rightarrow$	nuclear shapes and compressibility
C_3	$\Rightarrow$	nuclear radius

d. With ~250 parameters, get ± 0.1 MeV accuracy (0.0001 u)

3. Interpretation

a. ^{56}Fe Most Stable

Differentiate <BE> with respect to Z and A;
set to zero; gives minimum at ^{56}Fe

REASON: Competition between surface-energy losses (favors large A)
and Coulomb losses (favors low Z) balance at ^{56}Fe

REMEMBER THIS

b. Loss of binding at low A / Increase in Q value for fusion
Large surface energy loss at low A since most nucleons are on surface

c. Loss of binding at high A / Increase in Q value for fission
Coulomb repulsion competes with nuclear force

d. N/Z ratio > 1 for heavy nuclei

e. Fine Structure: Pairing

Problem: Which is more stable: $^{124}_{50}Sn$ $^{200}_{80}Hg$

Major Terms:

Surface Energy – small for both ($1/A^{1/3}$), favors ^{200}Hg slightly.

Coulomb Energy – large for ^{200}Hg ($80^2/50^2$) , favors ^{124}Sn.

Minor Terms:

Symmetry Energy – $^{200}_{80}Hg$: $(N\text{-}Z)^2 = 40^2$
$^{124}_{50}Sn$: $(N\text{-}Z)^2 = 34^2$; favors ^{124}Sn

Pairing Energy – No effect, both e-e.