

What did you learn in the last lecture?

### 3. Nuclear Reaction Energetics – Q values

Definition: Q is the energy RELEASED in a nuclear reaction, i.e. when two nuclei collide.

i.e., for  $A + B \rightarrow C + D + Q$

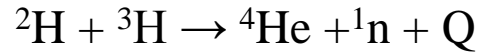
$$Q = \sum \Delta(\text{reactants}) - \sum \Delta(\text{products})$$

SIGN of Q

$$\left\{ \begin{array}{l} Q = + \quad \rightarrow \text{EXOTHERMIC} \\ Q = - \quad \rightarrow \text{ENDOTHERMIC} \end{array} \right\}$$

NOTE: A negative Q value can always be overcome by accelerating one of the reactants and converting kinetic energy to mass energy.

b. Example: Fusion power utilizes the following reaction



$$Q = \Delta({}^2\text{H}) + \Delta({}^3\text{H}) - \Delta({}^1\text{n}) - \Delta({}^4\text{He})$$

$$Q = 13.136 + 14.950 - 8.071 - 2.425$$

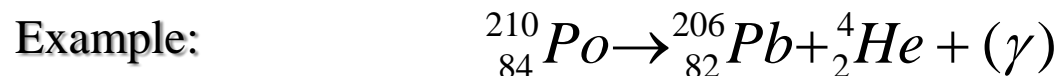
$$Q = 17.590 \text{ MeV}$$

Is this reaction exothermic or endothermic?

This energy appears as the kinetic energy of both  ${}^4\text{He}$  and neutron and can be converted to heat.

Nuclear Decays (an introduction) – see for example Ehmann and Vance Ch. 2

## Alpha Decay



Half-life ( $t_{1/2}$ ) of  ${}^{210}Po$  is 138.4d. What is the Q value of this reaction?

$$Q = \Delta({}^{210}Po) - \Delta({}^{206}Pb) - \Delta({}^4He)$$

$$Q = -15.969 - (-23.801 + (2.425)) = 5.407 \text{ MeV}$$

Where does this energy go?

$$P_{Pb} = -P_{He}$$

$$M_{Pb} V_{Pb} = -M_{He} V_{He}$$

$$V_{Pb} = -\frac{M_{He}}{M_{Pb}} V_{He}$$

$$\frac{1}{2} M_{Pb} V_{Pb}^2 + \frac{1}{2} M_{He} V_{He}^2 = 5.407$$

Substituting,

$$\frac{1}{2} M_{Pb} \left( \frac{M_{He}^2 V_{He}^2}{M_{Pb}^2} \right) + \frac{1}{2} M_{He} V_{He}^2 = 5.407$$

Rearranging,

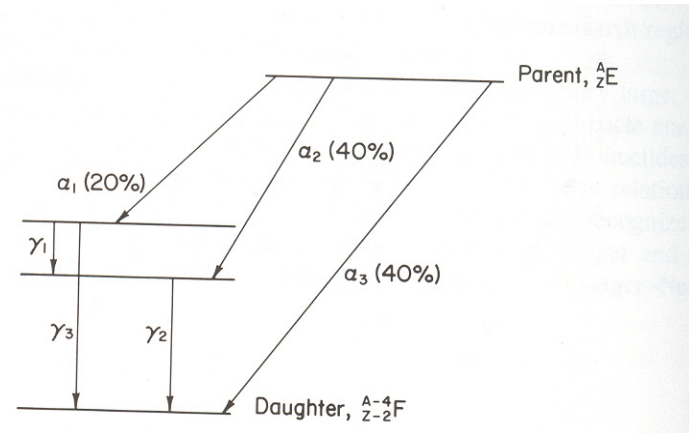
$$\frac{1}{2} M_{He} V_{He}^2 \left( \frac{M_{He}}{M_{Pb}} \right) + \frac{1}{2} M_{He} V_{He}^2 = 5.407$$

$$KE(He)\left(1 + \frac{M_{He}}{M_{Pb}}\right) = 5.407$$

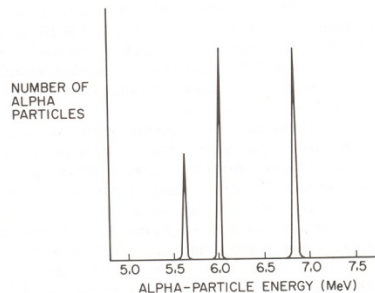
$$KE(He) = \frac{5.407}{1 + \frac{4.0026}{205.9805}}$$

$$KE(He) = 5.304 \text{ MeV}$$

The actual decay energy observed for the alpha particle is 5.304 MeV. In addition to the case where the parent nucleus decays to the ground state of the daughter, it is also possible for the parent nucleus to decay to an excited state of the daughter nucleus. Consider a general case,



Notice that in this generalized example, the parent nucleus E decays to the ground state of the daughter F, as well as its first and second excited state. The probability of each transition is called the branching ratio and is displayed in parentheses. As a result of this decay one would observe:



## Beta ( $\beta$ ) decay (an introduction)

There are three types of  $\beta$  decay.

- $\beta^-$  decay (net conversion of a neutron into a proton)
- $\beta^+$  decay (net conversion of a proton into a neutron)
- Electron capture (a nucleus captures an orbital electron to convert a proton into a neutron)

### a) $\beta^-$ decay :

GENERAL:  ${}^A_Z E \rightarrow {}^A_{Z+1} F + \beta^- + \bar{\nu} + (\gamma)$       atomic number increased by one  
and mass number is unchanged

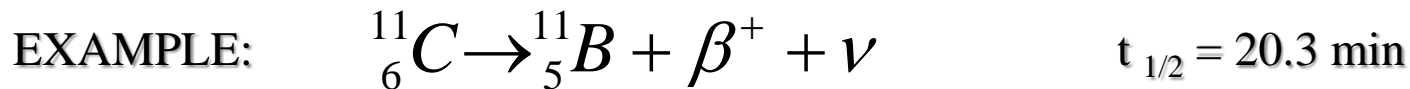
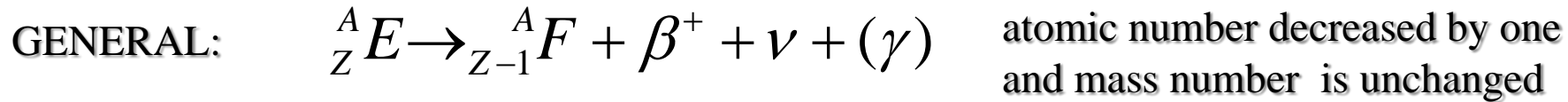
EXAMPLE:  ${}^{90}_{38} Sr \rightarrow {}^{90}_{39} Y + \beta^- + \bar{\nu}$        $t_{1/2} = 28.90 \text{ y}$

What is ? It is an **antineutrino**?

$\beta^-$  decay occurs for neutron-rich nuclides (when there is an “excess of neutrons”).

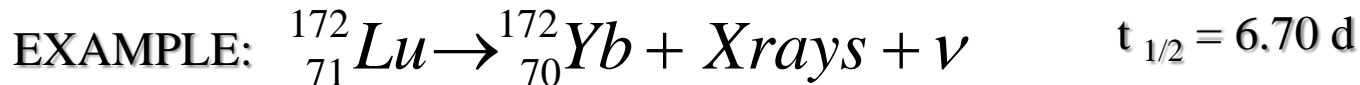
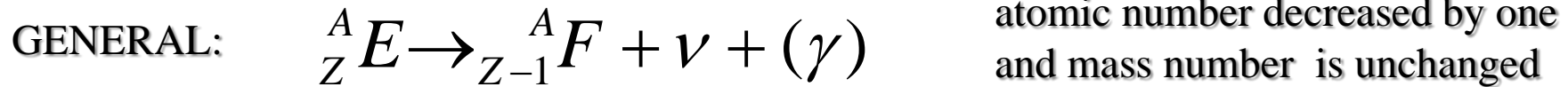
Neutron-rich relative to what?

b)  $\beta^+$  decay : (positron decay)



Positron decay occurs for proton-rich nuclides.

c) Electron capture (EC or  $\epsilon$ )



- Same daughter nuclide produced as in positron decay
- EC is the only possible decay mode to this daughter when the transition energy is less than 1.02 MeV (i.e.  $2m_e$ )
- When both modes can occur EC is favored in the case of high Z nuclei (inner electrons closer to nucleus) and the transition energy is low.
- Capture of a n=1 electron is called K capture, n=2 is called L capture etc.

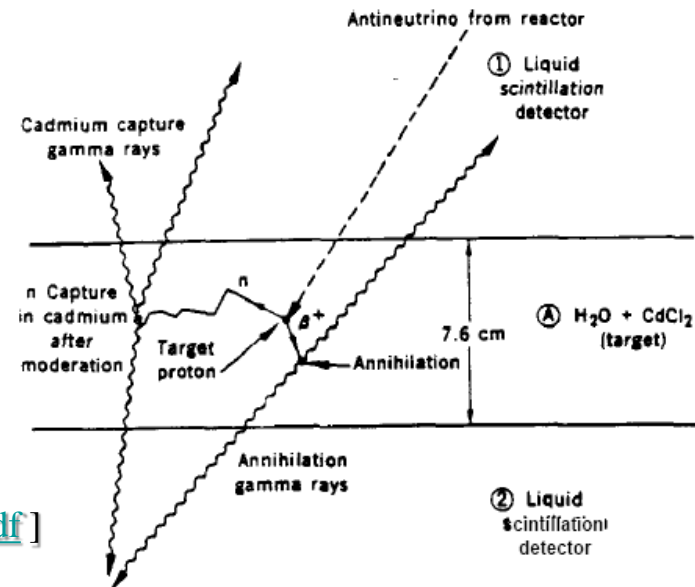
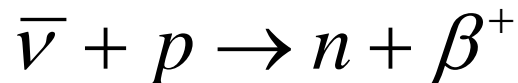


## What are these particles the neutrino and anti-neutrino?

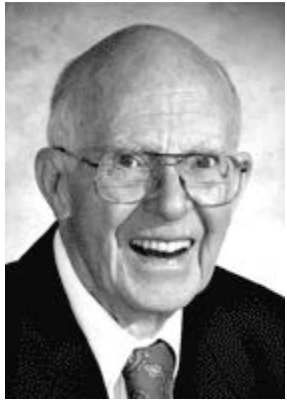
Early measurements of decay appeared to violate the conservation of momentum, leading Wolfgang Pauli to postulate that another particle, the anti-neutrino was involved.

The neutrino interacts so weakly with matter that Pauli stated “*I have done a terrible thing. I have postulated a particle that cannot be detected.*”

1953 Reines and Cowan detected antineutrinos through the following reaction.



[ See also Reine's 1995 Nobel prize lecture  
<http://nobelprize.org/physics/laureates/1995/reines-lecture.pdf> ]



## **Raymond Davis Jr. – Autobiography (2002 Nobel Prize in Physics)**

I was born in Washington, D.C. on October 14, 1914. My father was a photographer at the National Bureau of Standards. A self-educated man, he never finished high school, but, in his career at the National Bureau of Standards, he made many useful inventions, and eventually became chief of the Photographic Technology Section. His early influence led me in the direction of individual experimentation and designing my own apparatus. My mother, Ida Rogers Younger, was a native of the state of Virginia. She taught me to enjoy music, although she never succeeded in making me a performer. It was to please her that I spent several years as a choirboy, in spite of my inability to carry a tune. A bit later in life, I took pleasure in attending outdoor concerts at the Watergate, in the days before air traffic grew heavy enough to drown out the music.

**In early experiments, I attempted to detect neutrinos from a reactor, using the chlorine-argon detection method suggested by Bruno Pontecorvo (in 1946). In this method, a  $^{37}\text{Cl}$  atom reacts with a neutrino to make an  $^{37}\text{Ar}$  atom. Argon is a noble gas and is easy to separate chemically from a large amount of chlorine-rich solvent. It is radioactive with a half-life of 35 days and can be counted with a gas-filled proportional counter. A first attempt, exposing a 1000-gallon tank of carbon tetrachloride at the Brookhaven Graphite Research Reactor, failed to detect any signal, as the neutrino flux at this reactor was too small to affect a target of this size. Furthermore, a reactor emits antineutrinos, and the Pontecorvo method only detects neutrinos. It was not known at that time that the two particles were not identical. Later, I built larger experiments, using one of the Savannah River reactors as the neutrino source. I eventually set a limit on the neutrino flux that was a factor of 20 below the antineutrino flux measured by [Reines](#) and Cowan in their elegant experiment that won Fred Reines his Nobel Prize.**

Other early interests included the development, with Oliver Schaeffer, of a method of geological dating using  $^{36}\text{Cl}$  in surface rocks. With the later advent of accelerator mass spectrometry, this has become a useful tool in geochemistry, but our counting techniques were not sensitive enough to make the method work. We turned to measuring  $^{36}\text{Cl}$  in meteorites. Measuring the  $^{36}\text{Cl}$  radioactivity and the total accumulated decay product,  $^{36}\text{Ar}$ , in a meteorite allowed us to determine how long the meteorite had been exposed in space. Our interest in meteorite exposure ages continued for many years. We also worked on measuring cosmic-ray production of  $^{37}\text{Ar}$  and  $^{39}\text{Ar}$  in a variety of freshly fallen meteorites. Our greatest success in this work was with the Lost City meteorite. The track of this meteorite was photographed as it fell, allowing its orbit to be determined. Our measurement of radioactive argon isotopes allowed us to deduce the cosmic ray intensity gradient in the inner solar system. During the era of the moon landings, I was involved in measuring  $^{37}\text{Ar}$ ,  $^{39}\text{Ar}$ , tritium and  $^{222}\text{Rn}$  in lunar rocks and in the lunar atmosphere (trapped in the rock boxes brought back by the astronauts). During processing of the Apollo 12 samples, one of the glove boxes in Houston leaked and I had the interesting experience of being quarantined with the astronauts and a few other unlucky scientists for two weeks until it was clear that we were not infected with any lunar diseases.

**Following the Savannah River experiments, I began thinking about detecting neutrinos from the Sun.** The first step was a pilot experiment located 2,300 feet underground in the Barberton Limestone Mine, near Akron, Ohio. Observing neutrinos from the Sun had the potential of testing the theory that the hydrogen-helium fusion reactions are the source of the Sun's energy. In the 1950s, however, the proton-proton chain of reactions was believed to be the principal neutrino source, but this chain only emitted low-energy neutrinos, below the threshold of the chlorine-argon reaction.

**A new measurement of the nuclear reaction  $3\text{He}+4\text{He}\rightarrow 7\text{Be}+g$  by Holmgren and Johnston in 1958, suggested that one of the terminal reactions in the proton-proton chain would produce energetic neutrinos which could be measured by the chlorine-argon radiochemical method. Encouraged by these developments, and with the support of the Brookhaven National Laboratory and the U.S. Atomic Energy Commission, I built a much larger experiment in the Homestake Gold Mine in Lead, South Dakota. The detector itself consisted of a 100,000-gallon tank filled with perchloroethylene, a solvent most commonly used for dry cleaning of clothing. The experiment was located nearly a mile underground, at the 4850 foot level of the mine. Initially, we observed no solar neutrino signal and expressed our results only as upper limits. Subsequent refinements in technique and, particularly, in counting methods, continued over the years, producing a solar neutrino signal approximately one-third of the expected flux from the standard solar model calculated by John Bahcall. This was the genesis of the so-called "solar neutrino problem".**

**The solar neutrino problem caused great consternation among physicists and astrophysicists. My opinion in the early years was that something was wrong with the standard solar model; many physicists thought there was something wrong with my experiment. Years of measurements produced consistent answers and many tests showed that there were no problems with experimental procedures. Many distinguished physicists suggested explanations for the low solar neutrino flux that now seem fanciful. Trevor Pinch, a sociologist, made a study of how scientists responded to the solar neutrino problem. The disagreement between the measured solar neutrino flux and that predicted by the standard solar model was confirmed for energetic 8B neutrinos by the Kamiokande II experiment in the late 1980s and for the lower energy pp neutrinos by the gallium experiments GALLEX and SAGE in the middle 1990s. Only recently, observations at the Sudbury Neutrino Observatory (SNO) in the Inco Nickel Mine in Sudbury, Ontario, Canada, have indicated that, indeed, the total number of solar neutrinos emitted agrees with the standard solar model prediction, but that two-thirds of the neutrinos change in the course of their journey to the Earth into other flavors (m and t neutrinos), a phenomenon known as neutrino oscillation. Only electron neutrinos can be detected with the Cl-Ar radiochemical method.**

[From \*Les Prix Nobel. The Nobel Prizes 2002\*, Editor Tore Frängsmyr, \[Nobel Foundation\], Stockholm, 2003](#)

This autobiography/biography was written at the time of the award and later published in the book series [Les Prix Nobel/Nobel Lectures](#). The information is sometimes updated with an addendum submitted by the Laureate. To cite this document, always state the source as shown above.

[Taken from <http://nobelprize.org/physics/laureates/2002/davis-autobio.html> ]

# Particles and anti-particles

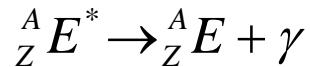
- Every particle has a corresponding anti-particle
  - Proton  $\leftrightarrow$  anti-proton
  - Neutron  $\leftrightarrow$  anti-neutron
  - Electron  $\leftrightarrow$  positron
  - neutrino  $\leftrightarrow$  anti-neutrino

The particle and anti-particle have the same mass but are opposite in charge. When a particle and anti-particle meet, they annihilate, that is their mass is converted into energy. Recall  $E=mc^2$ . For example, when a positron meets an electron the annihilation results in two gamma rays. To conserve momentum the gamma rays must be back to back. To also conserve energy, they must have the same energy. Each gamma therefore has an energy of 0.511 MeV, equal to the mass of the electron/positron. They are often called **annihilation gammas**. This distinguishing energy and angle relationship makes them easy to isolate from any other gammas.

**A real world example of this is the process of positron emission tomography (PET).**

## Nuclear Decay modes

**Gamma Decay**: Electromagnetic radiation corresponding to transition of nucleus from a higher excited state to a lower excited state. **It is always associated either with another radioactive decay or a nuclear reaction.**



Three modes of gamma decay:

- a. Pure gamma emission
- b. Internal conversion (IC)
- c. Pair production (PP)

a) Pure gamma emission : The gamma rays emitted by a nucleus in the gamma decay process are monoenergetic for each transition between energy levels. The gamma energies typically range from 2 keV to 7 MeV. Obviously energy must be conserved and momentum must be conserved. Thus, a small recoil energy/momentum is imparted to the daughter nucleus (can generally be ignored). Nevertheless, this energy is small so the energy of the gamma is very close to the energy of the transition.

b) Internal conversion (IC): The excited nucleus de-excites by transferring its energy to an orbital electron. This energy exceeds the binding energy of that electron so consequently that electron is ejected.



**The IC electrons are mono-energetic.** Their kinetic energy is equal to the energy of the transition minus the binding energy of the electron.

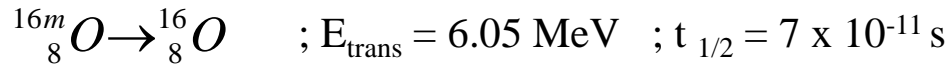
Internal conversion and pure gamma decay are competing processes!

Because IC decay results in a vacancy in an atomic orbital in the daughter nucleus, the electrons in the daughter nucleus “shuffle down” resulting in the emission of X-rays and Auger electrons.

As IC and pure gamma decay compete we can define the internal conversion coefficient ( $\alpha$ ):

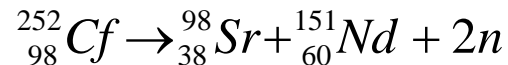
$$\alpha = \frac{\gamma_{decay}}{IC_{decay}}$$

c) Pair Production : For nuclear transitions with energies greater than 1.022 MeV, it is possible for the decay energy to directly produce an electron-positron pair which is then ejected from the nucleus. Recall  $E=mc^2$  where  $m= 2m_e$ . The total kinetic energy of the pair is equal to the difference between the transition energy and 1.022 MeV (which is  $2m_e$ ).

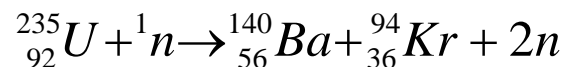


## Fission

**Spontaneous fission** is the process in which a heavy nucleus breaks into two smaller nuclei. From the curve of  $\langle BE \rangle$  we understand why this is energetically favorable.



Fission can also be **induced** by bombarding a heavy target nucleus with neutrons or charged particles thus exciting it.



Remember conservation of mass number and atomic number (charge)!



# Nuclear Sizes and Shapes

## Size Measurements

Perform relative to a standard length.

Atoms and Nuclei – microscopic: measurement: meter stick?

Criterion of applicability:      $\lambda \approx$  size of object

$$\lambda_{\text{photon}} = \frac{hc}{E_{\gamma}} = \frac{1.24 \times 10^3 \text{ fm}}{E_{\gamma} (\text{MeV})}$$

$$\lambda_{\text{particle}} = \frac{h}{p} = \frac{h}{mv} = \frac{28.7 \text{ fm}}{(A \times E (\text{MeV}))^{1/2}}$$

## Energies necessary for different probes

probe	<u>Atoms (<math>10^{-8}</math> cm)</u>	<u>Nuclei (<math>&lt;10^{-12}</math> cm)</u>
photon	$\sim 10$ keV ( $v \equiv c$ )	$\sim 100$ MeV ( $v \equiv c$ )
electron	$\sim 100$ eV ( $v \sim 0.1c$ )	$\sim 100$ MeV ( $v \approx c$ )
nucleon	$\sim 0.1$ eV ( $v \sim 10^5$ cm/s)	$\sim 10$ MeV ( $v \sim 0.1 c$ )

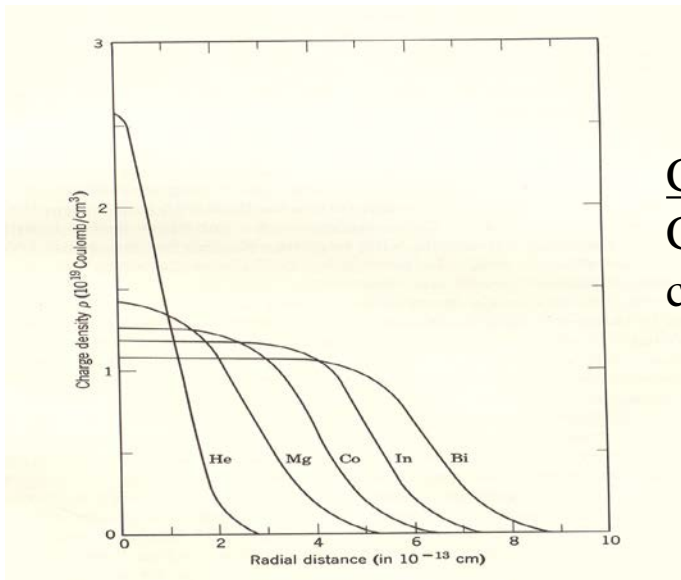
## **Nuclear Sizes( $r$ ) and Densities ( $\rho$ )**

Let's graph the density distribution of a basketball or ping pong ball.

How about the density distribution of a billiard ball?

Charge density distribution of a nucleus from electron scattering SLAC: 21 GeV  $e^-$ 's ;  $\lambda \sim 0.1$  fm

(to first order assume that this is also the matter distribution of the nucleus)



### CONCLUSION:

Central uniform density and diffuse surface (cloudy crystal ball)

## **Nucleus of A Nucleons**

### a. First Approximation: Uniform Density Sphere

$$\text{Volume } V \cong A \times V_{\text{nucleon}} = \left(\frac{4}{3}\right) \pi R^3 ; V_{\text{nucleon}} \approx \text{constant}$$

b. Liquid Drop Analogy: Assume all nucleons uniformly distributed for V

then,  $R = [(3A \bullet V_{\text{nucleon}})/4\pi]^{1/3} = r_0 A^{1/3}$  ;  $r_0$  is the nuclear radius constant.  
 $r_0 \approx 1.2 - 1.4$  fm (ALWAYS GIVEN)

Calculate the radius of  ${}^{216}_{84}\text{Po}$       Given  $r_0 = 1.40 \text{ fm}$

$$R = r_0 A^{1/3} = (1.40 \text{ fm})(216)^{1/3} = 8.40 \text{ fm} = 8.40 \times 10^{-13} \text{ cm}$$

b. The density distribution of real nuclei is described by a Woods-Saxon Shape (Fermi function)

$$\rho(r) = \frac{\rho_0}{1 + e^{-(r-R_{1/2})/d}}$$

$\rho_0$  = central density (  $\sim 2 \times 10^{14} \text{ g/cm}^3$  )

$R_{1/2}$  = half-density radius ;  $R_{1/2} = r_0 A^{1/3}$ ,  $r_0 = 1.07 \text{ fm}$

i.e., radius at which  $\rho = \rho_0/2$

$d$  = diffuseness ; distance over which  $\rho(r)$  decreases from  $0.90 \rho_0$  to  $0.10 \rho_0$   $d \sim 2.4 \text{ fm}$

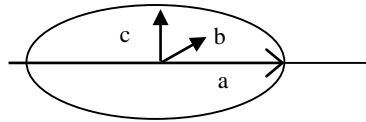
# Nuclear Shapes

1. Spherical : Near  $N$  or  $Z = 2, 8, 20, 28, 50, 82,$  and  $126$  (neutrons)

Magic  
numbers



2. Spheroidal : For nucleon numbers midway between magic number



i) Prolate:  $a > b = c \Rightarrow$  rugby ball

ii) Oblate:  $a < b = c \Rightarrow$  discus

3. Exotic Shapes

Octupole (pear-shaped) ;          fission (dumbell) ;

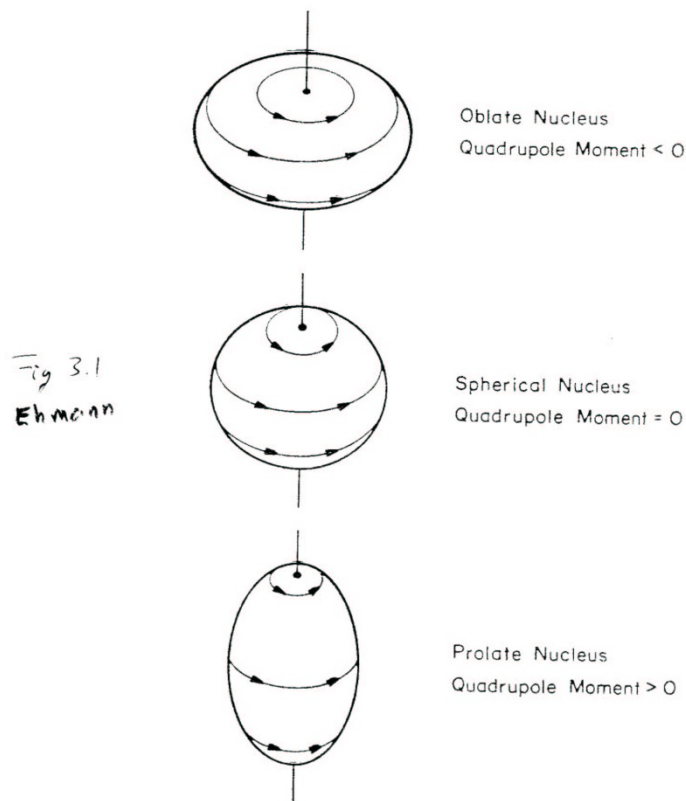


Figure 3.1. Shapes of the nucleus.

Motion of the charged particles (protons) within a nucleus represents a current just as the electrons moving through a wire do. Consequently a magnetic field is generated by the nucleus. This field is static (unchanging) and is called the magnetic moment of the nucleus. This magnetic moment tells us about the shape of the nucleus.

# Nuclear Models

## Philosophy and Difficulties

- Nuclear Force – no analytical expression
  - Many-Body Problem – no mathematical solution
- } ⇒ computer approximations

## MODELS

### Macroscopic Properties

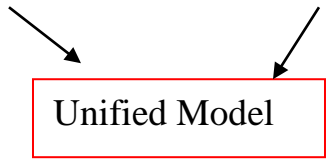
Energetics, Sizes, Shapes

Assumes all nucleons are alike  
(except charge)

### Microscopic Models

Spins, Quantum States, Magic Numbers

Assumes all nucleons are different



Unified Model

## Liquid Drop Model (Neils Bohr- 1940's)

Assumption:

The nucleus is a charged, nonpolar liquid drop,  
Chemical analogy: a cluster of Xe and Xe<sup>+</sup> atoms held  
together by Van der Waals attractions

## Justification

### 1. Nuclear Behavior: Similarities to liquid drop

- a. Force is short-ranged ; i.e., “sharp” boundary at surface
- b. Force is saturated; i.e. all nucleons in bulk of the liquid  
are bound equally, independent of radius
- c. Nucleus is incompressible at low temperatures – accounts for uniform density and constant  $\langle BE \rangle$
- d. Surface Tension – Surface nucleons lose binding ;  $\therefore$  spherical

### 2. Differences

- a. Few Particles ;  $A \lesssim 270$  vs  $10^{23}$
- b. Protons carry charge
- c. Two types of particles
- d. Result: microscopic properties exert significant influence and modify simple results.