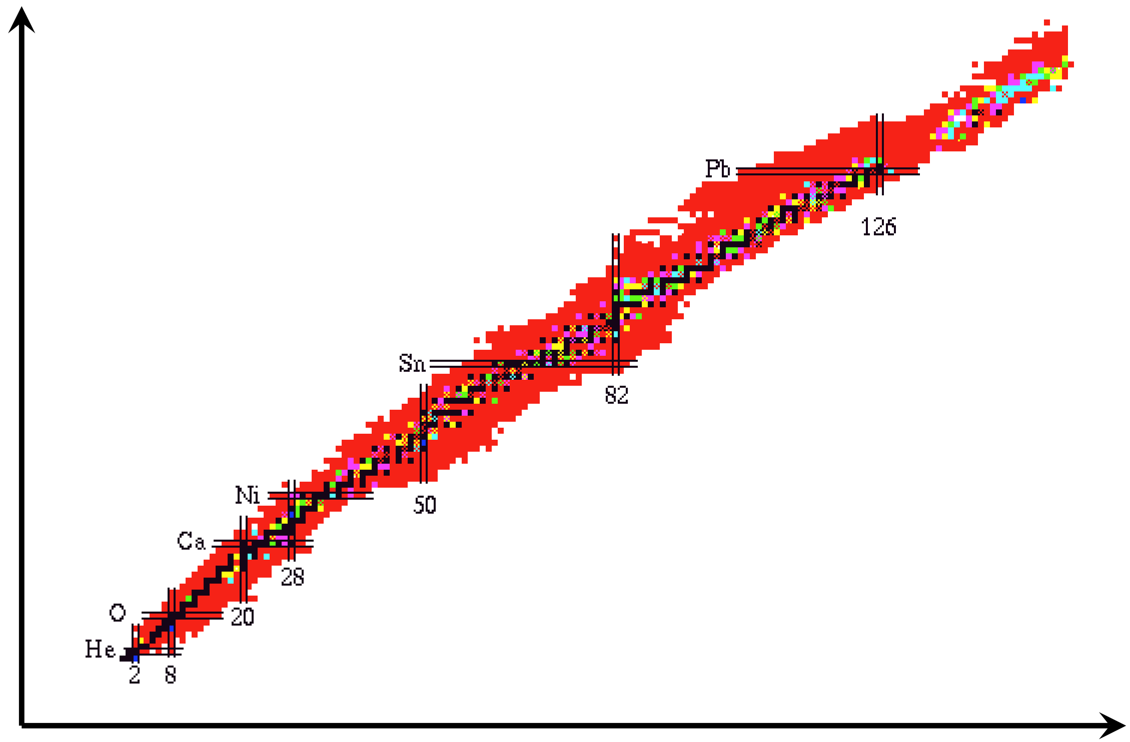
Radioactivity, Radiation and the Structure of the atom

What do you know (or can we deduce) about radioactivity from what you have learned in the course so far?

How can we learn about whether radioactive decay has occurred?

Chart of Nuclides: Not all combinations of neutrons and protons are allowed.

Atomic Number



Neutron Number

Why do some combinations of neutrons and protons occur and not others?

Binding energy

$E=mc^2$; mass and energy are equivalent

c= velocity of light = 2.99792×10^{10} cm/s

1 amu = 1.660×10^{-24}g

$E=(1.660 \times 10^{-24}\text{g})(2.99792 \times 10^{10}\text{cm/s})^2 = 1.492 \times 10^{-3}$ erg

1 eV = 1.602×10^{-12} erg

$\therefore$ 1 amu = 931.5 MeV

Alternatively one may say that c^2=931.494 MeV/u

Symbol	Description	Mass in g	Mass in MeV
m_p	Mass of proton	1.6726×10^{-24} g	938.256
m_n	Mass of neutron	1.6748×10^{-24} g	939.550
m_e	Mass of electron	9.1×10^{-28} g	0.511006

- Note that there is a factor of ~1830-1840 between the mass of an electron and a proton/neutron.
- Why is the mass of a proton less than the mass of a neutron?

Using mass tables

Define: **Mass Defect** = Δ ; This is sometimes called the **Mass Excess**.

$$\Delta = (M - A)c^2$$

units of Δ in MeV

Or

$$M = A + \frac{\Delta}{c^2}$$

Units of M in amu abbreviated u

<u>Example</u>:
M(^{4}He) = 4.002603 u, what is Δ?

$\Delta = (M\text{-}A)c^2 = (4.002603 - 4)u(931.494\ MeV/u) = 2.425\ MeV$

Likewise, if you know Δ you can calculate M.

Nuclear Wallet Cards

Isotope Z	El	A	Jπ	Δ (MeV)	T½, Γ, or Abundance	Decay Mode
6	C	10	0+	15.699	19.255 s *53*	ε
		11	3/2−	10.650	20.39 m *2*	ε
		12	0+	0.000	**98.89% *1***	
		13	1/2−	3.125	**1.11% *1***	
		14	0+	3.020	5730 y *40*	β−
		15	1/2+	9.873	2.449 s *5*	β−
		16	0+	13.694	0.747 s *8*	β−
		17		21.04	193 ms *13*	β− , β−n 32%
		18	0+	24.92	88 ms *+9−8*	β−
		18	0+	24.92	66 ms *+25−15*	β−n 19%
		19		32.8	49 ms *4*	β− , β−n 61%
		20	0+	37.6	14 ms *6*	β− , β−n 72%
		21		46.0s		
		22	0+	52.6s	>200 ns	

Taken From Nuclear Wallet cards, 5th ed. 1995 J.K. Tuli (posted on class Website)

Chemical Atomic Weight

$$\langle M_Z \rangle = \sum_i f_i M_i$$

- f_i = relative abundance
- M_i = mass of each isotope

<u>Example</u>:

$f(^{63}Cu)$ = 69.09 % ; $f(^{65}Cu)$ = 30.91 % ;

$M(^{63}Cu)$ = 62.92959 ; $M(^{65}Cu)$ = 64.92779

Problem: Calculate $<M_{Cu}>$.

Relativistic Effects

Relativity says that if we increase the velocity of a particle, its total mass increases.

This prediction is verified at accelerators.

For example, for a 200 MeV proton

$v=0.6c$; $M/M_0 = 1.25$

IN THIS COURSE WE WILL IGNORE RELATIVITY (except in a few special cases).

$\therefore$ use classical equations of motion:

$$E = \frac{1}{2} M_0 V^2$$

expression for the kinetic energy of a particle with rest mass M_0 and velocity V

$$p = M_0 V$$

expression for the momentum of a particle with rest mass M_0 and velocity V

These approximations are not bad as long as $v/c \leq 0.1$

Nuclear Binding Energies (What drives stability)

1. <u>General Definition</u> :

Mass is converted into potential energy which holds the system together.

Consequence for relative motions of particles in nucleus (Note that nucleons in nucleus are in constant motion.)

Examples –

- M(nucleus) < (ZMp + NMn)
- M(atom) < (M(nucleus) + ZMe)
- M(molecule) < Σ M (atoms)

The difference in mass is what we call the BINDING ENERGY. ($E=mc^2$)

2. Total Binding Energy: TBE

TBE is the mass converted into energy when a nucleus is formed from its constituent nucleons and electrons.

$$Z\,{}^{1}H + N\,{}^{1}n \rightarrow {}^{A}_{Z}X + \frac{TBE}{c^2}$$; mass balance; LHS in u

$$TBE = \left(ZM_H + NM_n - M({}^{A}_{Z}X) \right) c^2$$; energy balance; LHS in MeV

Substituting $M = A + \Delta/c^2$,

$$TBE = Z\Delta_H + N\Delta_n - \Delta({}^{A}_{Z}X)$$; analogous to the heat of condensation

Reversing the equation defines nuclear vaporization

TBE for the deuteron is ≈ 2.2 MeV

TBE for ^{238}U is ~ 2000 MeV

3. Average Binding Energy : <BE>

$$\langle BE \rangle = \frac{TBE}{A}$$

; this quantity is more instructive the TBE. It is analogous to the molar heat of condensation rather than the heat involved in condensing an arbitrary amount.

Example: ^{12}C

$6\,^{1}H + 6\,^{1}n \rightarrow {}^{12}C + TBE/c^2$

TBE = 6(7.289) + 6(8.071) – 0 = 92.160 MeV

$\langle BE \rangle$ = 92.160/12

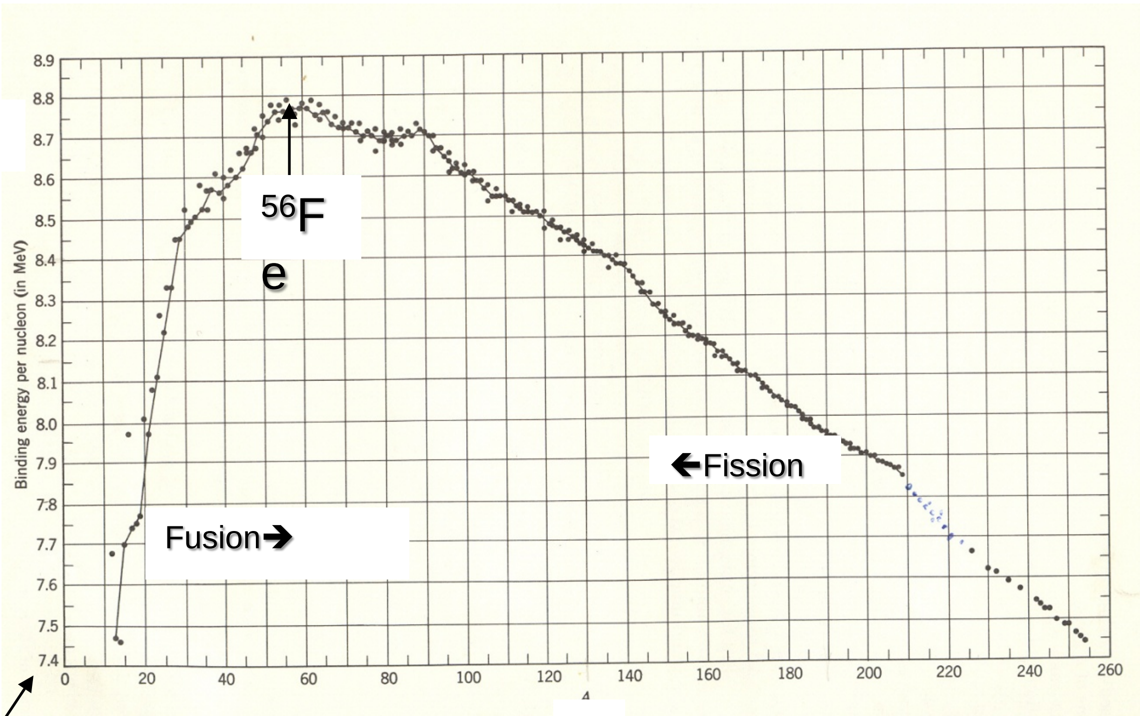
$\langle BE \rangle$ = 7.860 MeV

4. <u>Systematics of Average Binding Energies</u>

NOTE : LARGE <BE> IMPLIES HIGH STABILITY

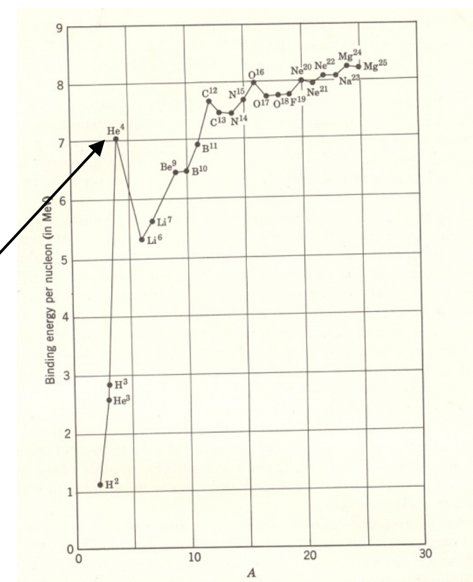
i.e. particles bound tightly together

<BE>



Note not zero on y-axis!

^{4}He is very tightly bound!



KEY POINTS

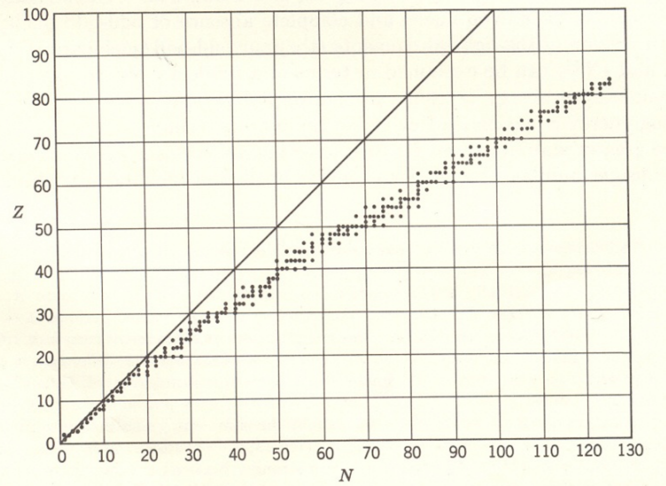


Fig. 2-5 The known stable nuclei on a plot of Z versus N. Note the gradual increase in the neutron-proton ratio; the 45° line corresponds to a neutron-proton ratio of unity.

• To first approximation <BE>~ 8 MeV i.e. relatively constant, acts like a nonpolar liquid

• N ≈ Z

• ^{56}Fe – most stable nucleus in nature

• Light nuclei -- <BE> increases with increasing A
Therefore ENERGY RELEASED DURING FUSION (Stellar Energy)
Fusion is the amalgamation of two nuclei to form a heavier nucleus.

• Heavy nuclei -- <BE> decreases with increasing A
∴ENERGY IS RELEASED DURING FISSION (Nuclear Reactors)
FISSION is the splitting of a heavy nucleus into two lighter nuclei.

• Fine structure
  - Peaks – quantum effects- shell structure
  - Odd-even variations – pairing effects

Nuclear Energetics

1. **Particle Binding Energies – B_i**

   Definition: The energy required to remove a particle from a nucleus
   (cf binding energy for e^-s in atoms)

2. **Cases:** In principle, either a nucleon or cluster of nucleons can be removed

   a. Proton Binding Energy (or Separation energy) -- B_p (or $S_p = - B_p$)

EQUATION:

$$ {}^{A}_{Z}X + \frac{B_p}{c^2} \rightarrow {}^{A-1}_{Z-1}Y + {}^{1}_{1}H $$

CALCULATION:

$$ B_p = \Delta(Y) + \Delta(H) - \Delta(X) $$

b. Neutron Binding Energy – B_n (or $S_n = -B_n$)

EQUATION:

$${}^{A}_{Z}X + \frac{B_n}{c^2} \rightarrow {}^{A-1}_{Z}X + {}^{1}_{0}n$$

CALCULATION:

$$B_n = \Delta({}^{A-1}X) + \Delta(n) - \Delta({}^{A}X)$$

c. Alpha Particle (${}^{4}He$) Binding Energy – B_α

EQUATION:

$${}^{A}_{Z}X + \frac{B_\alpha}{c^2} \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$

CALCULATION:

$$B_\alpha = \Delta(Y) + \Delta(\alpha) - \Delta(X)$$

d. etc. Could do same for ${}^{2}H$, ${}^{12}C$, ${}^{16}O$

e. Example: Calculate B_n for ^{12}Be

$$^{12}\text{Be} + B_n \longrightarrow \ ^{11}_{4}\text{Be} + \ ^{1}_{0}\text{n}$$

$$B_n = \Delta(^{11}\text{Be}) + \Delta(\text{n}) - \Delta(^{12}\text{Be}) \ \Rightarrow$$ Mass Table (Nuclear Wallet cards)

$$B_n = (20.174 + 8.071 - 25.007) \text{ MeV}$$

$$B_n = 3.168 \text{ MeV}$$

This means that if we supply 3.168 MeV to the ^{12}Be nucleus it will release a neutron.

3. Nuclear Reaction Energetics – Q values

Definition: Q is the energy RELEASED in a nuclear reaction, i.e. when two nuclei collide.

i.e., for $A + B \rightarrow C + D + Q$

$$Q = \sum \Delta(reactants) - \sum \Delta(products)$$

SIGN of Q

- Q = + ➔ EXOTHERMIC
- Q = - ➔ ENDOTHERMIC

NOTE: A negative Q value can always be overcome by accelerating one of the reactants and converting kinetic energy to mass energy.

b. <u>Example:</u> Fusion power utilizes the following reaction

$^{2}H + {}^{3}H \rightarrow {}^{4}He + {}^{1}n + Q$

$Q = \Delta(^{2}H) + \Delta(^{3}H) - \Delta(^{1}n) - \Delta(^{4}He)$

$Q = 13.136 + 14.950 - 8.071 - 2.425$

$Q = 17.590$ MeV

Is this reaction exothermic or endothermic?

This energy appears as the kinetic energy of both ^{4}He and neutron and can be converted to heat.

Nuclear Decays (an introduction) – see for example Ehmann and Vance Ch. 2

Alpha Decay

General form:

$$^{A}_{Z}E \rightarrow {}^{A-4}_{Z-2}F + {}^{4}_{2}He + (\gamma)$$

Example:

$$^{210}_{84}Po \rightarrow {}^{206}_{82}Pb + {}^{4}_{2}He + (\gamma)$$

Half-life ($t_{1/2}$) of ^{210}Po is 138.4d. What is the Q value of this reaction?

$Q=\Delta(^{210}Po)-\Delta(^{206}Pb)-\Delta(^{4}He)$

$Q= -15.969 - (-23.801+ (2.425)) = 5.407$ MeV

Where does this energy go?

$$P_{Pb} = -P_{He}$$

$$M_{Pb} V_{Pb} = -M_{He} V_{He}$$

$$V_{Pb} = -\frac{M_{He}}{M_{Pb}} V_{He}$$

$$\frac{1}{2} M_{Pb} V_{Pb}{}^{2} + \frac{1}{2} M_{He} V_{He}{}^{2} = 5.407$$

Substituting,

$$\frac{1}{2} M_{Pb} \left(\frac{M_{He}^{2} V_{He}^{2}}{M_{Pb}^{2}} \right) + \frac{1}{2} M_{He} V_{He}{}^{2} = 5.407$$

Rearranging,

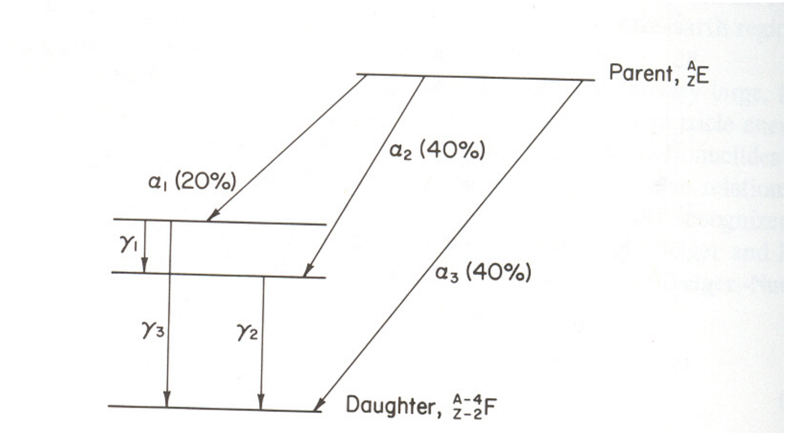
$$\frac{1}{2} M_{He} V_{He}^{2} \left(\frac{M_{He}}{M_{Pb}} \right) + \frac{1}{2} M_{He} V_{He}{}^{2} = 5.407$$

$$KE(He)(1+\frac{M_{He}}{M_{Pb}}) = 5.407$$

$$KE(He) = \frac{5.407}{1+\frac{4.0026}{205.9805}}$$

$$KE(He) = 5.304 \text{ MeV}$$

The actual decay energy observed for the alpha particle is 5.304 MeV. In addition to the case where the parent nucleus decays to the ground state of the daughter, it is also possible for the parent nucleus to decay to an excited state of the daughter nucleus. Consider a general case,



Notice that in this generalized example, the parent nucleus E decays to the ground state of the daughter F, as well as its first and second excited state. The probability of each transition is called the branching ratio and is displayed in parentheses. As a result of this decay one would observe: