Nuclear Reactions 2

D. Decay During Bombardment

1. Production of radioactive products that decay during bombardment is an important consideration in isotope preparation (secular equilibrium).

$$\frac{dN}{dt} = \underset{\text{Production Rate}}{R} - \underset{\text{Decay Rate}}{\lambda N}$$

2. Solution to Differential Equation

$$N = \frac{R}{\lambda}(1 - e^{-\lambda T})$$, where **T** = bombardment time

(assume R ≠ f (time) ; i.e., constant)

3. Saturation Factor: $1 - e^{-\lambda t}$

e.g. $T = t_{1/2}$, $N = (R/\lambda)(1\text{-}1/2) = (R/\lambda)(1/2)$

$T = 2t_{1/2}$, $N = (R/\lambda)(1\text{-}1/4) = (R/\lambda)(3/4)$; 50% more

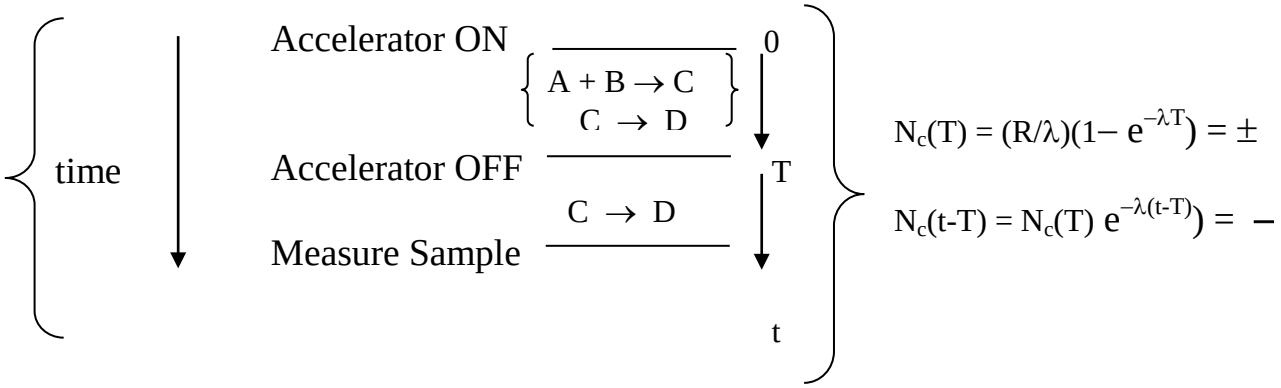
$T = 3t_{1/2}$, $N = (R/\lambda)(1\text{-}1/8) = (R/\lambda)(7/8)$; 25% more

That is, after one half-life bombardment time, reach a point of diminishing returns; cost of accelerator operation is constant.

4. Rearranging Above Result

$$N\lambda = R\,(1 - e^{-\lambda T}) = \frac{A}{c}$$

5. Chronology



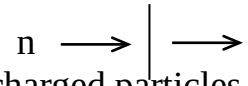
E. Neutron-Induced Reactions

No Coulomb barrier ($Z_p = 0$)

$\therefore$ **Only Q** influences energetics and **Q** is **always positive** ; $\therefore$ $E_{th} = 0$

$$\left\{ {}^{1}_{0}n + {}^{A}_{Z}X \rightarrow {}^{A+1}_{Z}X + Q \right\}$$ [**NOTE:** in reverse reaction $Q = B_n$

1. Neutron Beams:

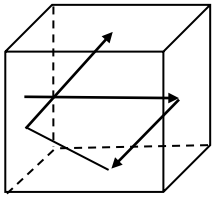


Same expression as for charged particles

$R = I\sigma x$

2. Nuclear Reactors: Common Situation

GEOMETRY: Neutron gas that permeates volume of solid, liquid or gas



(1) All target nuclei are accessible to neutrons: N_T

(2) Neutrons fill volume of sample (neutron gas)

n_n (neutrons/cm^3) $\times$ velocity (cm/s) = Φ , NEUTRON FLUX

$\Phi = n_n v$

i.e., higher the the neutron density and velocity, more likely to react.

$\therefore$ $\boxed{\text{Rate} = R = \sigma \Phi N_T}$ $\boxed{N\lambda = R(1 - e^{-\lambda T})}$

3. Neutron Capture Cross Section (Thermal neutrons)

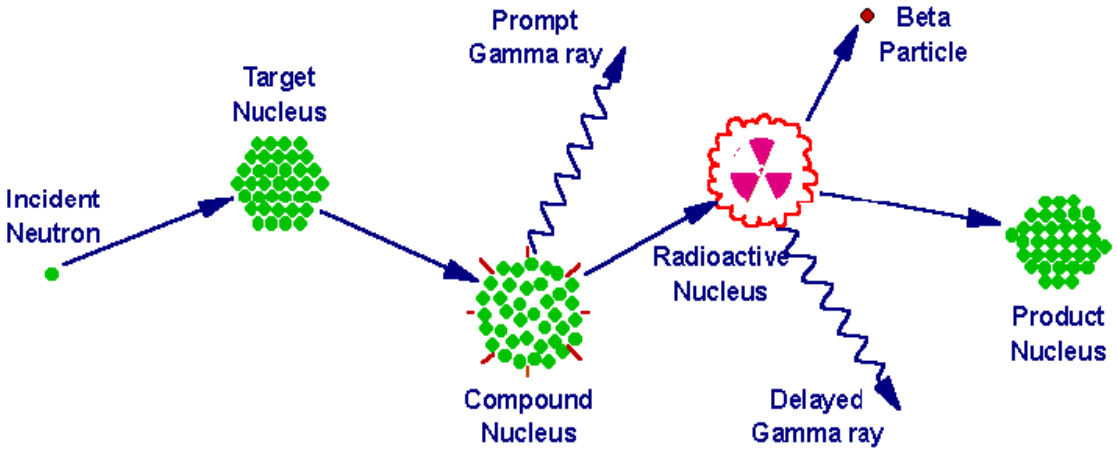
a. Typical range: $0.1 - 10^4$ b
e.g., $^{10}B(n,\alpha) = 3838$ b ; $^{113}Cd(n,\gamma) = 2.0 \times 10^4$ b

b. Resonance structures
$E_n \equiv$ levels in composite nucleus

c. Thermal Neutrons: 1/v Law

d. Comparison between charged particles and neutron-induced reactions

5. Practical Applications: NAA = Neutron Activation Analysis



- About 2/3 of periodic table accessible
  - Sensitivity down to picograms ; CAVEAT:

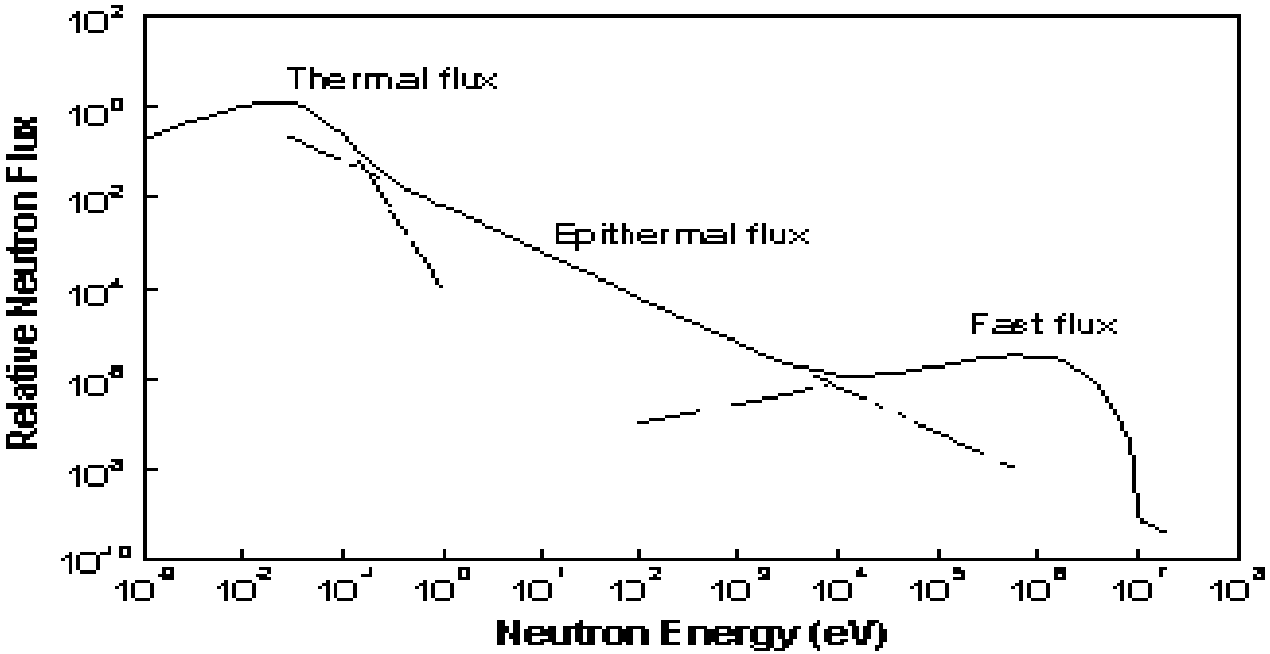
    NO CHEMICAL COMPOUND INFORMATION;
    ONLY ELEMENT YIELD AND ID

6. Areas of Application

(n,γ) reactions usually most important

a. Environment – atmospheric chemistry, marine samples, water pollution, toxic heavy metals (V, Cu/Cd, PCBs, Hg)
b. Cosmology – trace elements in meteorites and lunar samples
c. Geology – mineral analysis (^{252}Cf in situ); extinction theories (volcanoes vs. meteorites)
d. Medicine – Trace elements in body fluids
e. Criminology – gunpowder residues – sensitive to most elements in gunpowder
f. Art history – paint bases
g. Agriculture – herbicide and pesticide residue migration
h. Electronics – Metallurgy and Petroleum Engineering } Trace impurities

To measure trace levels you want a large intensity (flux) of the incident neutrons, this generally favors a reactor.



Thermal neutrons: E < 0.5 eV

Epithermal neutrons : 0.5 eV< E < 0.5 MeV

Fast neutrons: E > 0.5 MeV (generally fission neutrons; specialized technique fast NAA :**FNAA**)

Prompt gamma NAA (PGNAA) vs. Delayed Gamma NAA (DGNAA)

PGNAA is useful when the cross section is large.

DGNAA is often called conventional NAA or just NAA. Let's discuss this technique further

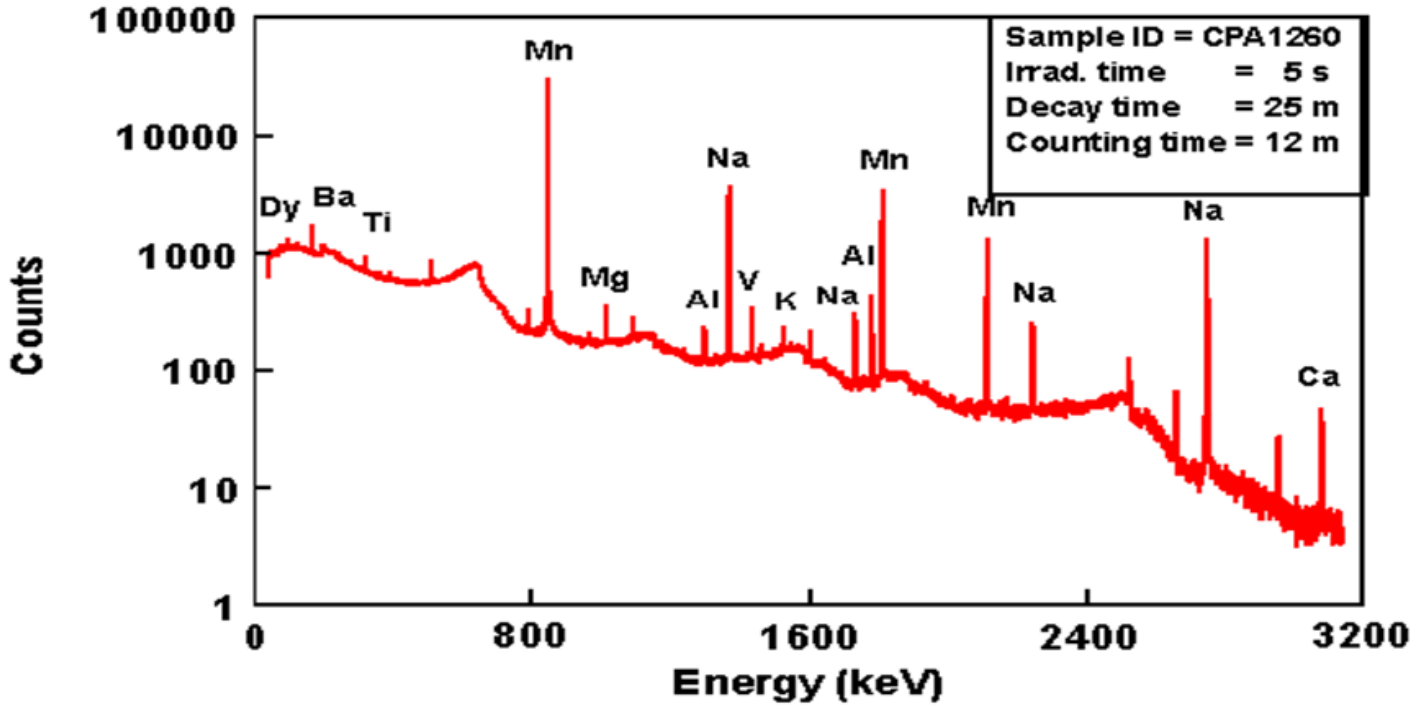


Fig. 3. Gamma-ray spectrum showing several short-lived elements measured in a sample of pottery irradiated for 5 seconds, decayed for 25 minutes, and counted for 12 minutes with an HPGe detector.

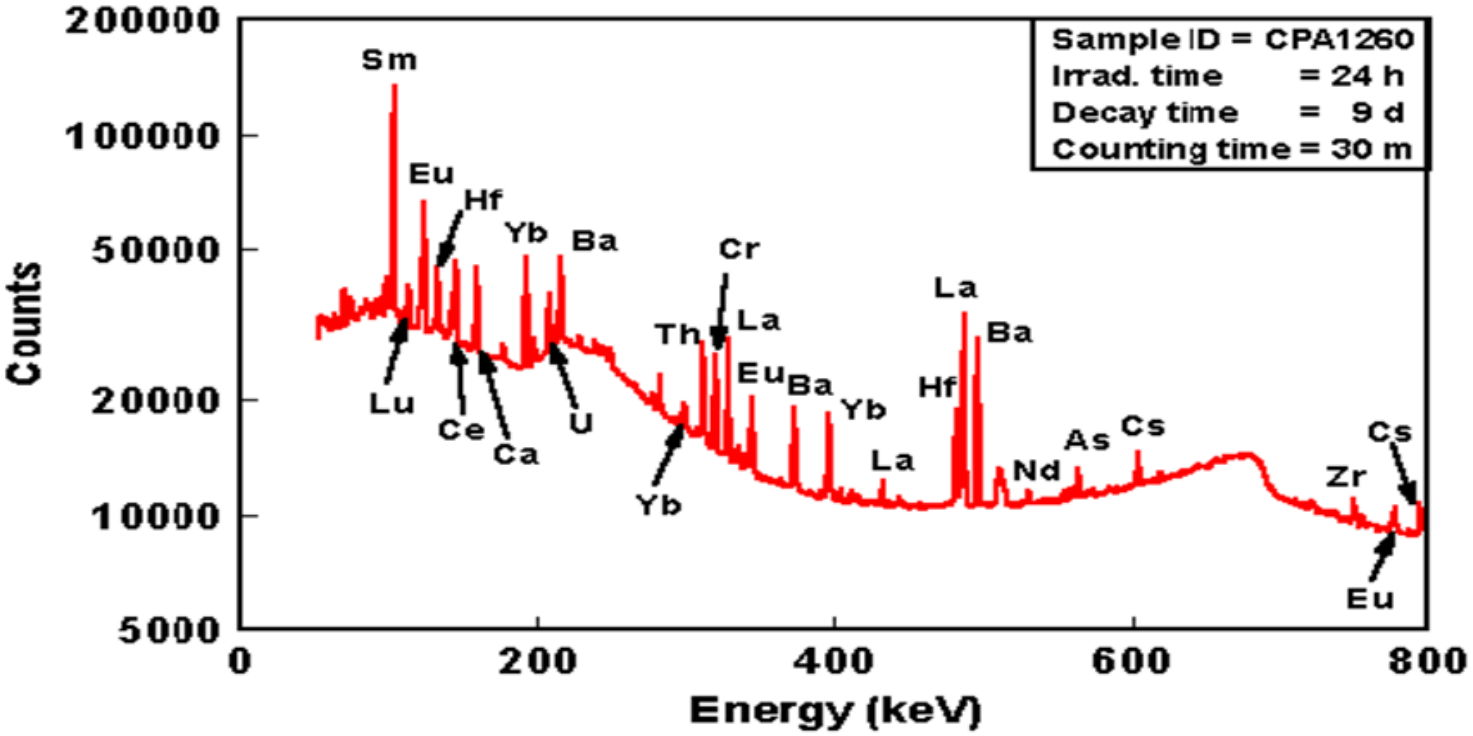


Fig. 4. Gamma-ray spectrum from 0 to 800 keV showing medium- and long-lived elements measured in a sample of pottery irradiated for 24 hours, decayed for 9 days, and counted for 30 minutes on a HPGe detector.

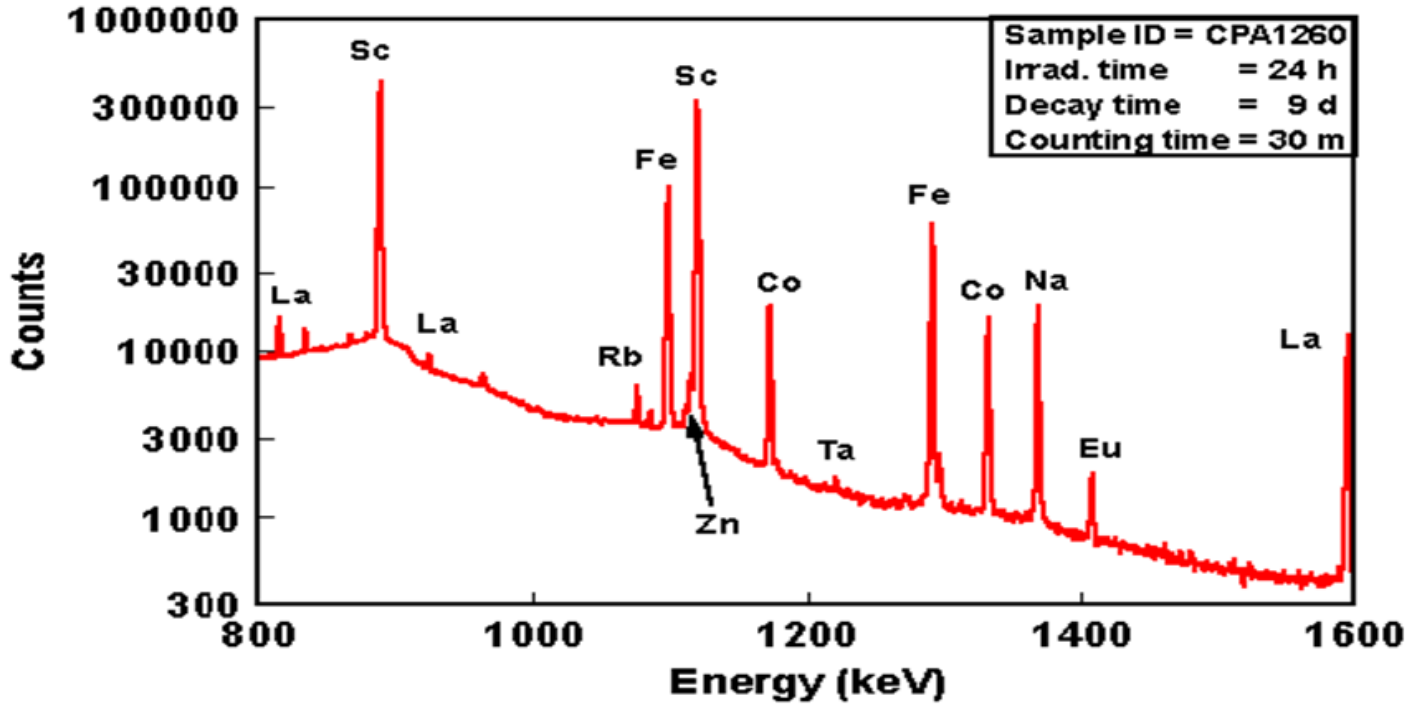


Fig. 5. Gamma-ray spectrum from 800 to 1600 keV showing medium- and long-lived elements measured in a sample of pottery irradiated for 24 hours, decayed for 9 days, and counted for 30 minutes on a HPGe dectector.

7. Sample Problem: What is the sensitivity (in grams) for Cr analysis with neutron activation analysis?

a. Step I: Select the **best product** isotope for **detection.**

$^{50}Cr(4.35\%)\ ;\ \sigma(n,\gamma) = 15.96\ b \rightarrow {}^{51}Cr(27.7\ d\ EC) + \gamma$

$^{52}Cr(83.79\%)\ ;\ \sigma(n,\gamma) = \text{———} \rightarrow {}^{53}Cr(stable)$ X

$^{53}Cr(9.50\%)\ ;\ \sigma(n,\gamma) = \text{———} \rightarrow {}^{54}Cr(stable)$ X

$^{54}Cr(2.36\%)\ ;\ \sigma(n,\gamma) = 0.3866 \rightarrow {}^{55}Cr(3.55\ min\ \beta^-)$

^{50}Cr or ^{54}Cr ?

^{50}Cr: larger abundance, larger σ
half-life a bit long

^{54}Cr: β^- easier to detect than EC ;
half-life short

BUT ^{50}Cr EC goes to excited state and emits γ

$\therefore\ {}^{50}Cr \Rightarrow$ DETECT ^{51}Cr

b. Step II: Irradiate sample, remove from reactor and count

- Sample preparation is important ; cleanliness and thermal stablilty
- Count γ-rays if possible – unique fingerprint, penetrate matter readily. Also removes necessity of chemical separations (yield loss) and handling (contamination).

c. <u>Sensitivity – Optimum</u>

- $\Phi = nv = 1.0 \times 10^{16}$ n/cm^2-s (usually 10-100 times less)
  $T_{irradiation} = 10.0$ d
  Detection limit = 0.10 dps for ^{51}Cr (background)

- If decay time is substantial between removal from reactor and counting, must do $N = N_0 e^{-\lambda t}$ correction

  e.g., wait 5.0 days

  $A(^{51}Cr)_{t\text{-}T} = A(^{51}Cr)_T \, e^{-\lambda t}$

  $0.10 \text{ dps} = A(^{51}Cr)_T \, (e^{-0.693(5.0\text{d})/27.7\text{d}})$

  $$A(^{51}Cr)_T = \frac{0.10 \text{ dps}}{0.882} = 0.113 \text{ dps}$$

i.e., Must have a counting rate of 0.113 dps at end of irradiation if counting rate is to be 0.10 dps when measurement takes place

- $N_T(^{51}Cr) = (R/\lambda)(1 - e^{-\lambda T})$

quantity of interest

$$R = \sigma \, N(^{50}Cr)\Phi = (15.96 \times 10^{-24} \text{ cm}^2)(N[^{50}Cr])(1.0\times10^{16} \text{ n/cm}^2\text{–s})$$

$$N_T(^{51}Cr)\lambda(^{51}Cr) = R \, (1 - e^{-0.693(10.0 \text{ d}/27.27 \text{ d}}) = R(1 - 0.779) = 0.221 \, R$$

$$0.221 \, R = 0.113 \text{ dps}$$

$$(0.221)(15.96 \times 10^{-24} \text{ cm}^2)(1.0\times10^{16} \text{ n/cm}^2\text{–s})N(^{50}Cr) = 0.113 \text{ dps}$$

$$N(^{50}Cr) = (0.113)/(0.221)(15.96)(10^{-8}) = 3.84 \times 10^6 \text{ atoms}$$

- N(Cr) = N(^{50}Cr)/abundance = $3.84 \times 10^6/0.0435 = 8.8 \times 10^7$ atoms

- wt (Cr) = n × g-at-wt = $= \frac{8.8 \times 10^7}{6.02 \times 10^{23}} \times 51g = 7.4 \times 10^{-15}\, g$

8. Centers of NAA (one is Univ. of Missouri, Columbia)

See http://www.missouri.edu/~glascock/naa_over.htm for an overview of NAA.
http://www.ncnr.nist.gov/instruments/nactanal.html NIST, Center for Neutron Research
http://ares.jsc.nasa.gov/Labs/neutronAAlab.htm, NASA
http://reactor.engr.wisc.edu/naa.html, University of Wisconsin, Nuclear Reactor Lab.
http://www.chem.tamu.edu/services/naa/, Texas A & M,
http://geology.cr.usgs.gov/facilities/gstr/, US Geological Survey $30/sample

and MANY more

9. Major restrictions: (1) can't do HCNO
(2) No molecular information

IV. Products of Nuclear Reactions

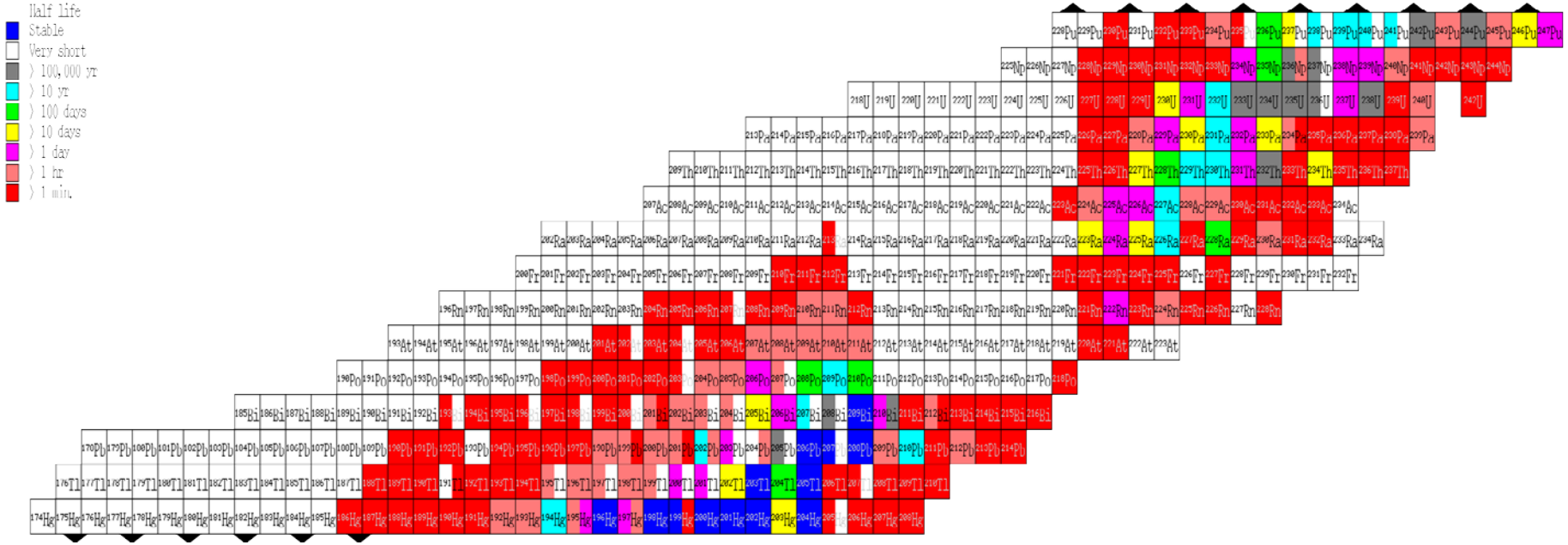
A. Excitation Functions

1. Definition: Cross section dependence on bombarding energy
2. Importance: (1) Testing nuclear reaction theory
   (2) Selecting proper bombarding energy for producing a given isotope in a nuclear reaction.

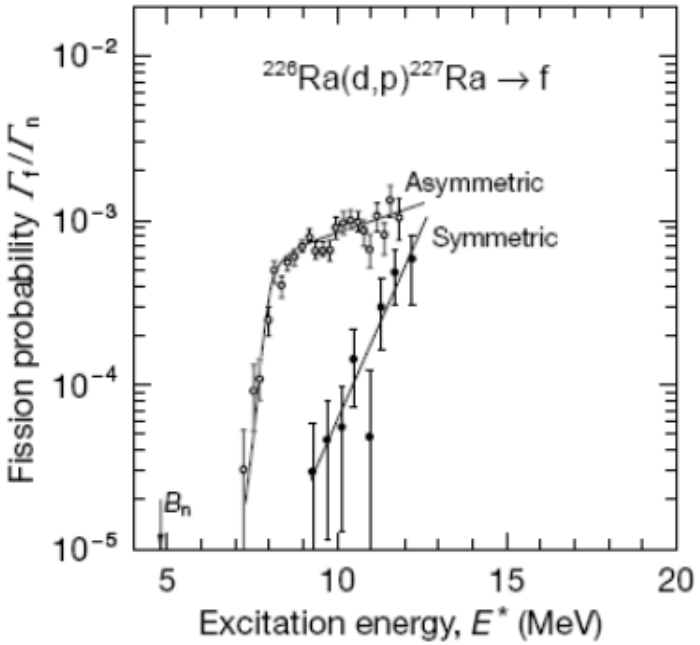
3. KNOW HOW TO READ AXES
   σ(a,b) compilations ; National Data Center @Brookhaven Natl. Lab (nuclear chemist's Beilstein)

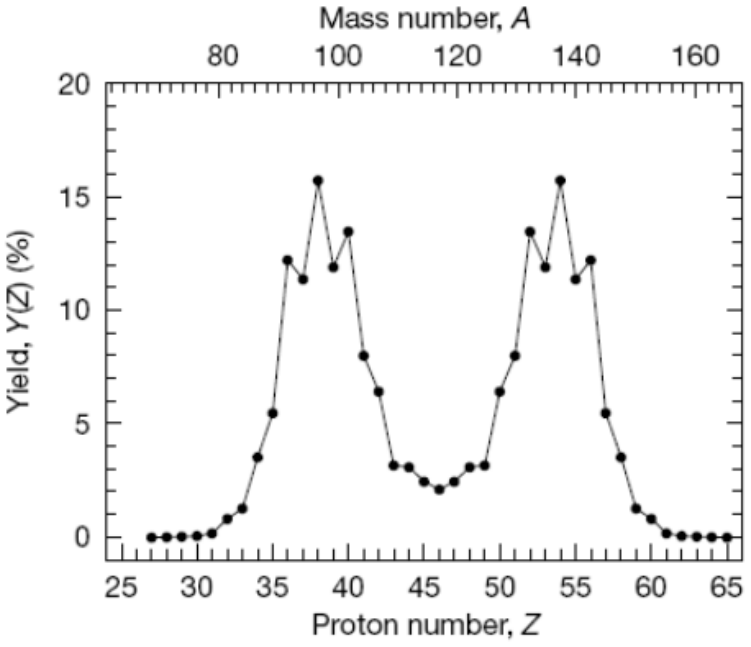
4. Emitted Particles

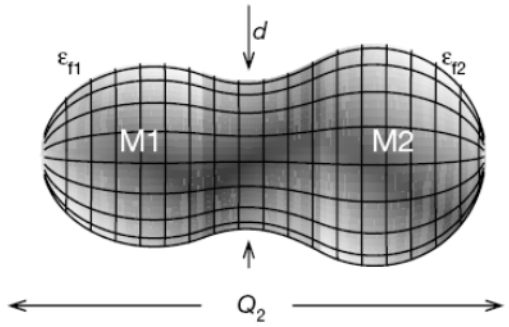
	Low bombarding energy	High Energy
Light targets	p, n, α, γ	everything Q allows
Heavy targets	n, fission, γ	everything Q allows



Fission (Figures below taken from Moller et al., Nature 409 (2001) 785



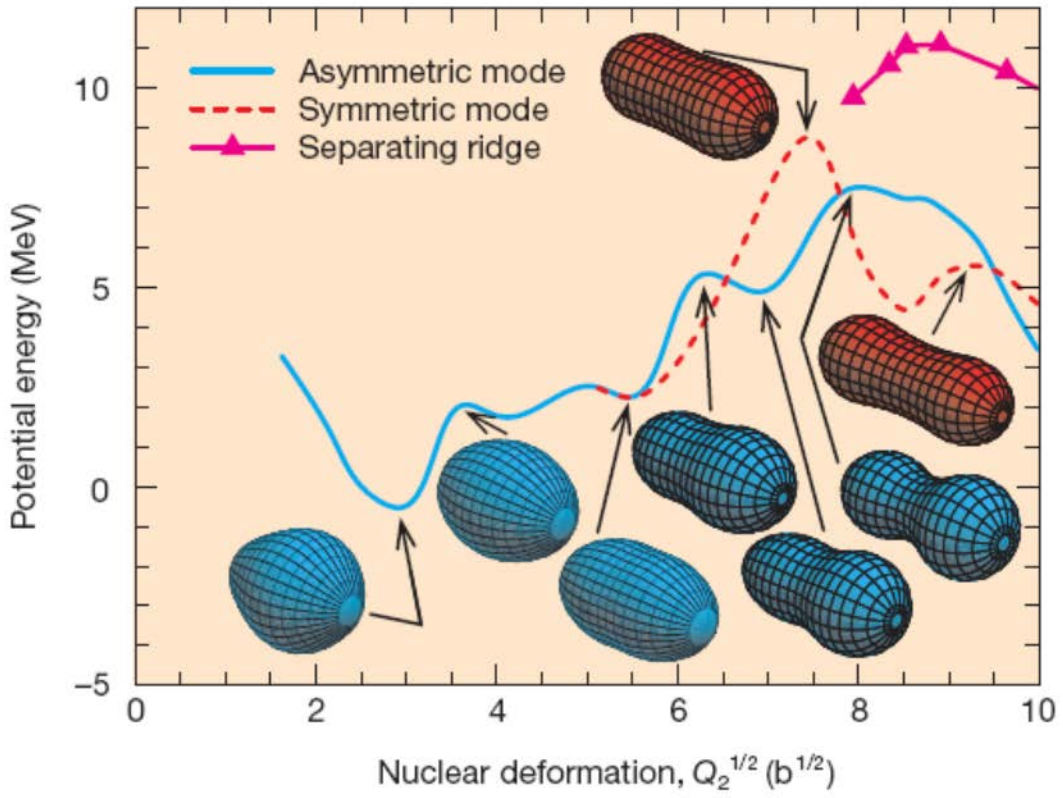




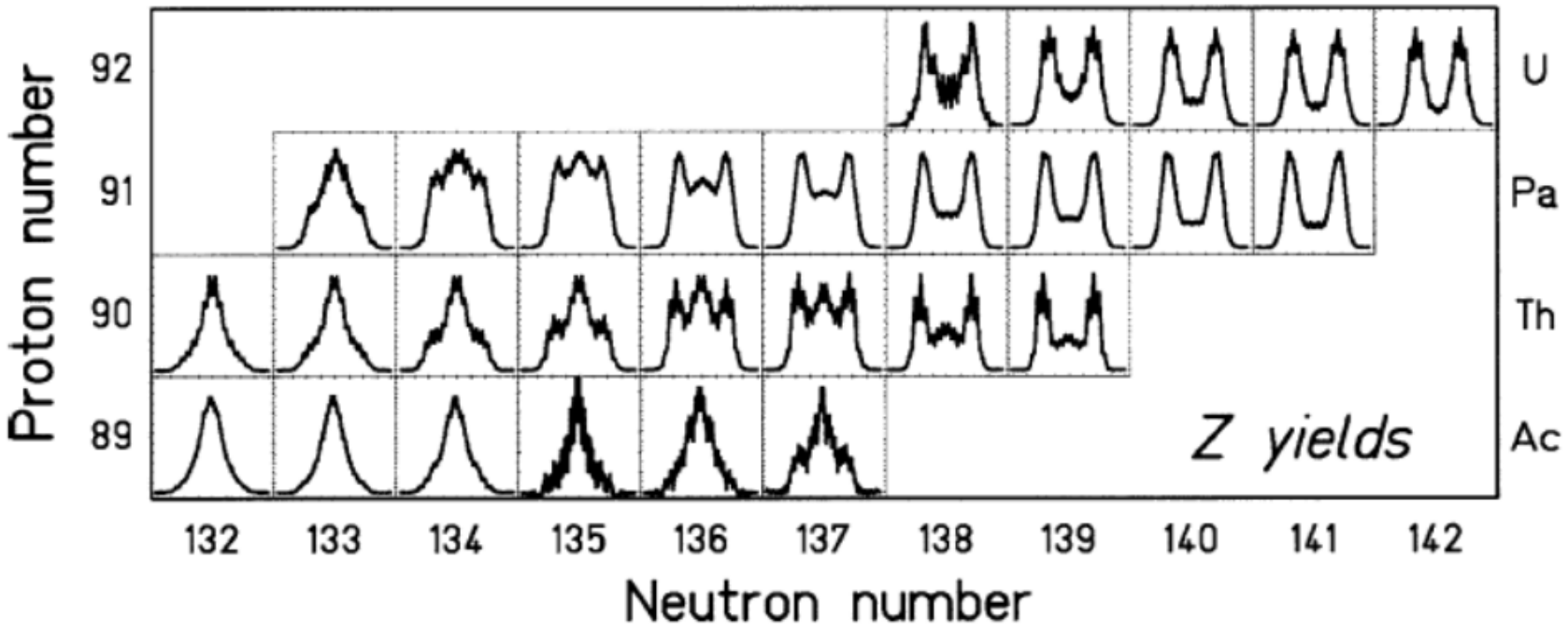
41	Q_2	Elongation (fission direction)
⊗		
20	α_g	(M1–M2)/(M1+M2) Mass asymmetry
⊗		
15	ε_{f1}	Left fragment deformation
⊗		
15	ε_{f2}	Right fragment deformation
⊗		
15	d	Neck

⇒ 2 767 500 grid points – 156 615 unphysical points

⇒ 2 610 885 physical grid points



The above quantities are measurable quantities. See for example K.-H. Schmidt et al., Nucl. Phys. A 693, (2001) 169.



The Z distributions shown above were measured for a radioactive nuclide accelerated to 1 GeV/A