# Nuclear Reactions

| | | | |
|---|---|---|---|
| **RELEVANCE**: | Basic Research – | reaction mechanisms | |
| | | structure | |
| | | exotic nuclei | |
| | Applications: | analytical tool | – neutron activation analysis |
| | | astrophysics | – origin of elements |
| | | space radiation effects | – satellites, Mars landing |
| | | materials science | – $T_c$(high) superconductors |
| | | medical therapy | – tumor treatment |
| | | nuclear power | – 25% of U.S. electricity |

DEFINITION: { A collision between two nuclei that produces a change in the nuclear composition and/or energy state of the colliding species ; i.e., $2^{nd}$ order kinetics }

# 1. Nomenclature

## A. Constituents

1. Projectile: nucleus that is accelerated ($v > 0$) ; $0 \rightarrow 2 \times 10^{12}$ eV

   a. neutrons – reactors
   b. light ions – $A \leq 4$: AGS (NY), FNAL (IL)
   c. heavy ions – $A > 4 \rightarrow U$: ANL (IL), MSU (MI), TAMU (TX), RHIC (NY)
   d. electrons (and photons) – SLAC (CA), CEBAF (VA), MIT (MA)
   e. exotic beams: $\pi$, K, $\overline{p}$ : FNAL
   f. radioactive beams: MSU, ANL (IL) HHJRF (TN)

2. Target: fixed ($v = 0$), usually

   But now have colliding beams, RHIC, FNAL

3. Products

   ANYTHING PERMITTED BY CONSERVATION LAWS.

   Note: since projectile kinetic energy can be converted into mass, only limit on mass-energy is beam energy.

## B. Notation:

Target [projectile, light product(s)] heavy product

e.g., ${}^{7}_{3}\mathrm{Li} + {}^{208}_{82}\mathrm{Pb}$

$\rightarrow$ ${}^{212}\mathrm{At} + 3n$ : ${}^{208}\mathrm{Pb}({}^{7}\mathrm{Li}, 3n){}^{212}\mathrm{At}$

$\rightarrow$ ${}^{197}\mathrm{Au} + {}^{18}\mathrm{C}$ : ${}^{208}\mathrm{Pb}({}^{7}\mathrm{Li}, {}^{18}\mathrm{C}){}^{197}\mathrm{Au}$

## C. Energetics: Q-values revisited

1. $Q = \sum\Delta(\text{reactants}) - \sum\Delta(\text{products})$ = available energy

2. Example:

${}^{208}\mathrm{Pb}({}^{7}\mathrm{Li}, \gamma)$ reaction: $Q = \Delta({}^{208}\mathrm{Pb}) + \Delta({}^{7}\mathrm{Li}) - \Delta({}^{215}\mathrm{At}) - \Delta(\gamma, )$

$Q = -21.759 + 14.908 - (-1.263) - 0$

$\boxed{Q = -5.589 \text{ MeV}}$ ENDOTHERMIC

Accelerate ${}^{7}\mathrm{Li}$ to 5.589 MeV, no reaction. WHY?

## II. Energetic Conditions for Reaction

### A. Energetic Threshold: $E_{th}$ & Excitation Energy: $E^*$

1. DEFINITION: $E_{th}$ is the minimum projectile energy necessary to satisfy mass-energy and momentum conservation (i.e., compensate for –Q)

2. Derivation:

a. Apply conservation Laws

projectile + target → composite nucleus → products

- Mass-energy: $\boxed{E_p + \Delta_p + \Delta_t = \Delta_{CN} + E_{CN} + E^*}$

$E_p$ = projectile kinetic energy

$E_T = 0$

$E^*$ = Excitation energy;

When $E^* = 0$
$E_p = E_{th}$

$$E = \frac{p^2}{2m};\ p = \sqrt[2]{2mE}$$

- Linear Momentum: $\vec{p}_p + \vec{p}_T = \vec{p}_{CN}$

$$\sqrt{2M_pE_p} \qquad 0 \qquad \sqrt{2M_{CN}E_{CN}}$$

OR $\qquad M_pE_p = M_{CN}E_{CN}$

$$\approx \boxed{A_pE_p = A_{CN}E_{CN}}$$

b. Combining

$$E_p + \Delta_p + \Delta_t = \Delta_{CN} + (A_p/A_{CN})E_p + E^*$$

$$E_p - (A_p/A_{CN})E_p = \Delta_{CN} - \Delta_p - \Delta_T + E^*$$

$$E_p (1 - A_p/A_{CN}) = E^* - [\Delta p + \Delta_T - \Delta_{CN}]$$

$$E_p \frac{A_{CN} - A_p}{A_{CN}} = E^* - Q$$

$$E_p (A_T/A_{CN}) = E^* - Q$$

- IF $E^* = 0$ (minimum), then $E_p = E_{th}$ and

$$\boxed{E_{th} = (A_{CN}/A_T)(-Q)}$$

- IF $E^* > E_{th}$ $\boxed{E^* = (A_T/A_{CN})\, E_p + Q}$

c. Example: $^{208}Pb(^{7}Li, \gamma)^{215}At$ ; $Q = -5.589$ MeV

$$E_{th} = -Q\left(\frac{A_{CN}}{A_T}\right) = -(-5.589\text{ MeV})(215/108) = \boxed{5.777\text{ MeV}}$$

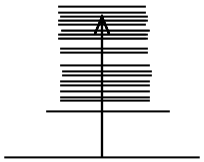
$$E_{CN} = E_{th} - E_p = 5.777 \quad -5.589 \quad = \boxed{0.188\text{ MeV}}$$

i.e., of total of 5.777 MeV, 5.589 MeV goes into mass ($M = E/c^2$) and 0.188 goes into kinetic energy of the composite nucleus.

d. <u>Example</u>: $E_p = 40.0$ MeV,

$$E^* = \left(\frac{208}{215}\right) 40.0 \text{ MeV} - 5.589 = 33.1 \text{ MeV}$$

- This energy is converted into heat



Fermi Gas model: $T \propto \sqrt{E^*/A}$

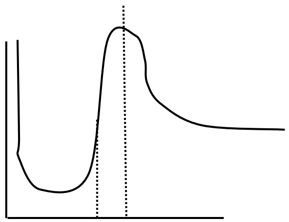
$$\left\{ 1 \text{ MeV} \approx 10^{10}\ ^\circ\text{K (kT)} \right\}$$

e. Accelerate to 5.777 MeV
Still no reaction. WHY?

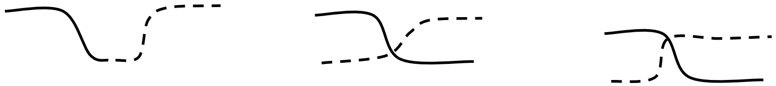
## B. Coulomb Barrier – Charged Projectiles Only

1. <u>Nucleus-Nucleus Charge Repulsion</u>

$$\left\{ V_{coul} = \frac{(Z_p e)(Z_T e)}{d} = \frac{Z_p Z_T e^2}{R_p + R_T} \right\}$$



2. <u>Potential Energy Surface</u>
Nuclear Reactions begin to occur when <u>**tails**</u> of nuclear matter distributions overlap



Bottom line: $r_0$ for reactions is greater than for potential well.

3. Net Result

$$\left\{ V_{Coul}^{cm} = \frac{1.44\, Z_p Z_T \text{ MeV} - \text{fm}}{r_0 (A_p^{1/3} + A_T^{1/3})} = \boxed{\frac{0.90\, Z_p Z_T}{(A_p^{1/3} + A_T^{1/3})} \text{ MeV}} \right\}$$

$r_0 = 1.60$ fm

4. Momentum Conservation

CN must carry off some kinetic energy; same correction as for Mass-Energy Conservation (Q-value)

$$\boxed{V_{Coul}^{lab} = \left(\frac{A_{CN}}{A_T}\right) V_{Coul}^{cm}}$$

5. Example $^{208}\text{Pb}(^{7}\text{Li}, \gamma)\, ^{215}\text{At}$

$$V_{Coul}^{lab} = \left(\frac{215}{208}\right) \frac{(82)(3)(0.9)}{(208^{1/3} + 7^{1/3})} = \boxed{29.1 \text{ MeV}}$$ **NOW THINGS HAPPEN**

6. Diffuse nuclear surface fuzzes precision of Coulomb barrier

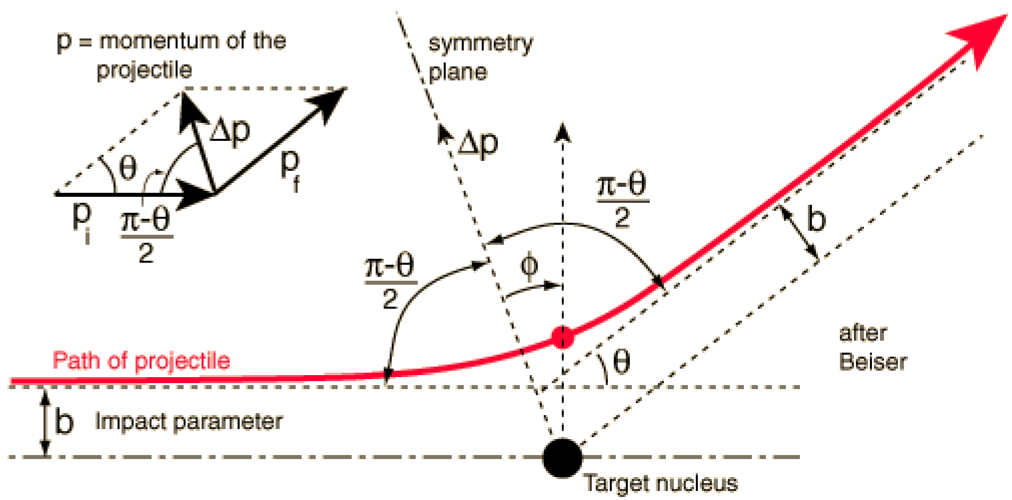
$\therefore$ $E_{th}$ is an **EXACT** condition

$V_{Coul}$ is an APPROXIMATE condition

## C. Centrifugal Barrier – NOT A MINIMUM CONDITION

### Alpha Scattering Geometry

The scattering of the alpha particle by the central repulsive Coulomb force leads to a hyperbolic trajectory. From the scattering angle and momentum, one can calculate the impact parameter and closest approach to the target nucleus.




Rotational energy = $\frac{\ell(\ell+1)\hbar^2}{2I}$ where $|l| = mvb$ and $I = \mu r^2$ where μ is the reduced mass of the system.

**The rotational energy is not available for reaction!**

## D. Summary of Energetic Factors

1. $E_p \geq E_{th}$: Mass-Energy Conservation: ABSOLUTE CONDITION – 1st law

2. $E_p \gtrsim V_{Coul}^{lab}$ : Charge Repulsion Constraint – BARRIER PENTRATION
   Probability low below this energy

3. $E_p \geq E_{rot}$ : No constraint since $\ell = 0$ is always possible.

4. IN GENERAL

   $\left\{ V_{Coul} > E_{th} \right\}$ ; except for very light nuclei and neutral projectiles,
   e.g., neutrons

## III. Reaction Probability: The Second-Order Rate Law

<u>Probability</u> $\equiv$ <u>$\sigma$</u> = <u>cross section</u> [SAME FOR CHEMICAL Rx]

### A. Schematic Picture

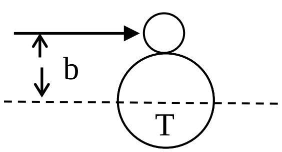
1. ON-OFF nature of nuclear force suggests a simple geometric model:

<u>Touching Spheres Model</u>

- <u>IF</u> projectile and target touch, REACTION ; $b \leq R_p + R_T$
- <u>IF</u> projectile and target don't touch , NO REACTION; $b > R_p + R_T$

2. Bottom Line: Probability is proportional to cross-sectional area

$$\text{Area} = \pi(R_P + R_T)^2 = \sigma$$




$$\boxed{\sigma_R = \pi\, r_0^2\, (A_p^{1/3} + A_T^{1/3})^2}$$

TOTAL REACTION CROSS SECTION

$r_0 \approx 1.60$ fm

3. Unit: $\boxed{1 \text{ barn} = 1 \times 10^{-24} \text{ cm}^2}$ = 1b

## B. Definitions

1. Sequential Process

| **COLLISION STAGE** | | **DECAY STAGE** |
|---|---|---|
| (entrance channel) | | (exit channel) |
| $\tau \sim 10^{-21} - 10^{-23}$ s | $\xrightarrow[\text{Equilibration}]{\text{Mass-Energy}}$ | $\tau \sim 10^{-15} - 10^{-21}$ s |
| $\sigma$= REACTION CROSS SECTION | | $\sigma(a,b)$= PRODUCTION CROSS SECTION |
| Collision Probability | | Probability of forming a given product (many may be possible) |

2. $\sigma_R = \sum \sigma(a,b)$

i.e., Sum of all possible production $\sigma$'s equals the total reaction $\sigma$

3. Example

40-MeV $^{4}He + {}^{232}Th \rightarrow {}^{236}U^{*} \rightarrow$ products

| | | | |
|---|---|---|---|
| $\sigma(\alpha, pn)$ | = | 0.010 b | $^{234}_{91}Pa$ |
| $\sigma(\alpha, 2n)$ | = | 0.080 b | $^{234}U$ |
| $\sigma(\alpha, 3n)$ | = | 0.245 b | $^{233}U$ |
| $\sigma(\alpha, 4n)$ | = | 0.125 b | $^{232}U$ |
| $\sigma(\alpha, f)$ | = | 1.190 b | fission |
| $\sigma_R$ | = | $\boxed{1.650\text{ b}}$ | = $\underline{\sum\sigma(\alpha, x)}$ |

## C. Cross Section Measurements: Nuclear Reaction Rates

1. Review of Biomolecular Rate Law

$A + B \rightarrow C + D$ [Elementary Reaction – only kind for nuclei]

$$\left\{ \text{Rate} = -\frac{d\,[A]}{dt} = k\,[A][B] \right\}$$

i.e., a second-order rate process

a. [A][B] factor: Collision Probability

Defines collision geometry ; e.g., two gases or two liquids, molecular beam + gas, etc

b. k = rate constant = probability of reaction **IF** collision occurs.

$k = f\,(\Delta H_a, T, \text{structure, etc.})$

2. Nuclear Case: Charged-Particle-Induced Reactions

$k = \sigma$ ; $[A][B] = n_p n_T$ (projectile nuclei × target nuclei); definition of n is geometry dependent

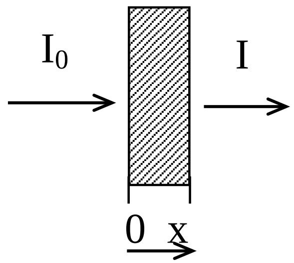
R = Rate = Number of Product nuclei/time $\sigma(a, b)$ **OR**

Number of Reactions/time $\sigma_{total}$

$\sigma$ = cross section – must be given

3. Thick Target: General Case

For thick targets, nuclear reactions remove a significant fraction of the beam:




$I \ \alpha \ \ n_p$ = number of projectiles/time

$I_0 - I$ = rate of reaction = $dI/dx$

$x$ = target thickness

$\therefore \ R = (-dI/dx) = \sigma \, n_t \, I \, dx$ , where $\mathbf{n_t} = \rho \, N_0/(\text{g–at. wt.})$

i.e., number density ($N/cm^3$)

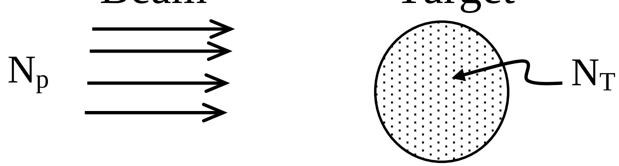
$$\int_{I_0}^{I} \frac{dI}{I} = -\sigma n_t \int_0^x dx$$

$I = I_0 \, e^{-n\sigma x}$ Beer-Lambert Law

4. Thin Target Case

GEOMETRY: Beam + infinitely thin target [thin = no shadowed nuclei]

Beam Target = fixed (i.e., v = 0); Area of target > Area of beam



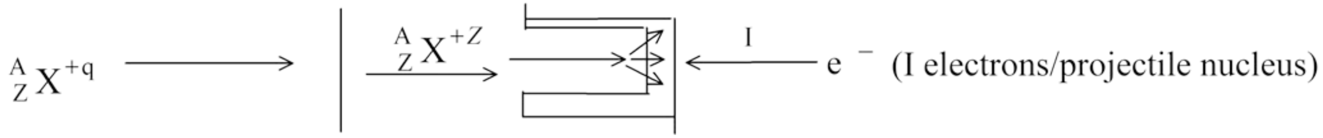

Collision Probability: $(N_p/t)(N_T/\text{area})$

Reaction probability: $\sigma = f(Q, V_c, I\pi, \text{etc.})$

Thin target Result:

Rate = $I\sigma(n_t x)$

a.




Projectile Beam  Target  Faraday Cup

To count projectiles, measure current *i* collected in Faraday cup

**I** = *i* / particle charge q

**I** is measured in Amps, since charged particles

q = $Z_p$ e (ion charge)

e.g., 10nA ${}^{12}C$ beam (stripped of electrons)

$1nA = 6.28x10^9 e^- / s$

$10nA = 6.28x10^{10} e^- / s$

However, each ${}^{12}C$ has a charge of +6.

$$\therefore \# \, {}^{12}C/s = \frac{6.28x10^{10} e^- / s}{6e^- / {}^{12}C} = 1.05x10^{10} \; {}^{12}C/s = \mathbf{I}$$

b. $(n_t x) = \# t \arg etatoms / cm^2$ = density x thickness

c. TOTAL RATE R = Number of Reactions/unit time

R = I$\sigma$($n_t$x)

5. Problem: What is the production rate of ${}^{266}_{106}Sg$ if a 100 μg/cm$^2$ target of ${}^{248}_{96}\mathrm{Cm}$ is bombarded with a 1.0 μA beam of ${}^{22}$Ne ions?

$\sigma$ (${}^{22}$Ne, 4n) = 1.0 nb

$R = I\sigma(n_t x)$

$$\sigma = (1.0 \times 10^{-24}\ \mathrm{cm}^2)(10^{-9}) = 1.0 \times 10^{-33}\ \mathrm{cm}^2$$

$$(n_t x) = \frac{(100 \times 10^{-6})\mathrm{g}}{248\ \mathrm{g/mole}} \frac{(6.02 \times 10^{23}\ \mathrm{atoms/mole})}{\mathrm{cm}^2} = 2.43 \times 10^{17}\ \mathrm{atoms/cm}^2$$

$$I = 1.0\ \mu\mathrm{A} \times 6.28 \times 10^{12}\ e^-/\mu\mathrm{A}\ /\ (10\ e^-/\mathrm{Ne}) = \boxed{6.28 \times 10^{11}\ \mathrm{Ne/s}}$$

$$R = (1.0 \times 10^{-33}\ \cancel{\mathrm{cm}^2})\left(\frac{2.43\times10^{17}}{\cancel{\mathrm{cm}^2}}\right)(6.28 \times 10^{11}/\mathrm{s}).$$

$$\boxed{R = 1.5 \times 10^{-4}/\mathrm{s} = 0.54/\mathrm{hr}}$$

$\sigma$ (${}^{22}$Ne, f) = 2.5 b

NOTE: 2 fragments/fission

$$\boxed{R = 7.6 \times 10^{5}/\mathrm{s}}$$

i.e., humongous fission background