C. Alpha Decay Probability

1. Energetics: \( Q_\alpha \) positive for all \( A > 140 \) nuclei

2. Range of Measured Half-Lives (~\( 10^{44} \))
   \[ 10^{16} \text{ y} > t_{1/2} > 10^{-21} \text{ s} \]

3. Why \( \alpha \)?
   a. Proton & Neutron Emission: \( Q_p, Q_n \) are negative near valley of beta stability (peak of peninsula); Thermodynamically forbidden
   b. Other Nuclei; e.g. \( ^{12}\text{C}, ^{16}\text{O} \) …
      - \( Q(^{12}\text{C}), Q(^{16}\text{O}) \) positive; therefore possible
      - Probability is low (i.e., \( t_{1/2} \) is long) -- \( P(^{14}\text{C})/P_\alpha \sim P^{-10} \)
      \[ ^{232}\text{Th} \rightarrow ^{14}\text{C} + ^{218}\text{Po} \] (Exotic decay mode)

FACTS

- 11.7 MeV \( \alpha \) particles from \( ^{212m}\text{Po} \) are the highest energy alphas from a radioactive source
- 2.0 MeV alphas from Sm are the among lowest energy alphas from a radioactive source
- Most alpha particles from radioactive sources fall in the range of 4-8 MeV.
- Associated with this narrow range in energy is the enormous range in half-life noted above.

WHY?
4. Coulomb Barrier Penetration

What is the relevant potential for the nucleus? We can consider this by examining the approach of an alpha particle from infinity (microscopic reversibility). If the collision is head-on (dead center) there are two parts to consider the attractive nuclear potential and the repulsive Coulomb potential.

\[ V(r) = V_{\text{nuclear}} + V_{\text{Coulomb}} \]
α escapes via quantum-mechanical tunneling through the Coulomb barrier.

a. Electrostatic Repulsion Energy for Two Charged Spheres:

\[ V_{\text{Coulomb}} = \frac{(Z_1e)(Z_2e)}{R} = 1.44 \frac{Z_1Z_2}{R} \text{MeV.fm} \]

where

\[ Z_1 = 2 \]
\[ Z_2 = Z_{\text{recoil}} \]
\[ R = R_1 + R_2 = r_0 \left( A_1^{1/3} + 4^{1/3} \right) \]

b. Example: \( ^{220}_{102}\text{No} \) let \( r_0 = 1.4 \text{ fm} \)

\[ V_{\text{Coulomb}} = \frac{1.44(2)(100)}{1.4(4^{1/3} + 216^{1/3})} = 25.5\text{MeV} \] 

Typical of most heavy nuclei.
c. \(Q_\alpha \leq 10 \text{ MeV} \); \[\therefore V_{\text{coul}} >> Q_\alpha \] and can't escape classically

d. Relative Barriers (approximate)

\[V_{\text{coul}}(\alpha): V_{\text{coul}}(^{12}\text{C}): V_{\text{coul}}(^{16}\text{O}) \approx 2:6:8\] (since Z & R of recoil \(\approx\) constant)
\[\therefore\] heavier fragments have much higher (and thicker) hill to punch through

5. Probability – \(P_\alpha\)

a. Tunneling Probability:

\[P_\alpha \propto (P_{\text{formation}})e^{(Q_\alpha - V_{\text{Coulomb}})}\]

\[P_\alpha \propto \frac{1}{t_{1/2}(\alpha)}\]

b. \(P_\alpha\) favored by:

(1) large \(Q_\alpha\) and low \(V_{\text{coul}}\) or small \(Q_\alpha - V_c\)

\[\therefore t_{1/2}(\alpha) \propto 1/P(\alpha) \propto e^{-(Q_\alpha - V_c)}\]

or \(\log t_{1/2} \propto V_c - Q_\alpha\)
D. Applications/Environment

1. $^{241}\text{Am}$ (458y) — smoke detectors
2. $^{238}\text{Pu}$ (88y) — remote sensing devices; power sources
3. $^{222}\text{Rn}$ (3.8d) — natural radioactivity — health effects
4. $^{226}\text{Ra}$ (1620y) — cancer therapy
5. ($^{210}\text{Po}$) (138d) — power sources
6. Dating tags — U, Th, Sm
Beta-Decay: Neutron-Proton Transformations

Connects nuclei that are isobars

**NEUTRON EXCESS**

Negatron Decay

\[ n \rightarrow ^1H + \beta^- + \bar{\nu} \]

**PROTON EXCESS**

Positron/electron capture Decay

\[ ^1H \rightarrow ^1n + \beta^+ + \nu \]

\[ ^1H + e^- \rightarrow ^1n + \nu \]

\( Z_A = \text{most probable} \)
A. Beta-decay Half-lives

1. Factors that Influence Rate
   a. **WEAK NUCLEAR FORCE**: 
      
      \[ t_{1/2}(\beta^\pm) \geq 10^{-3} \text{ s} \] 
      
      ∴ SLOW !!! 
      \[ t_{1/2}(\beta^\pm) \geq 10^{-3} \text{ s} \] 
      (cf \( \geq 10^{-21} \text{ s} \) for α)
   b. Large \( Q_\beta^- \) ⇒ Short half-life
   c. Structure: spin and parity changes 
      \( (\Delta I \& \Delta \pi) \) retard rates

2. Example

\[ ^{24}_{11}\text{Na} \rightarrow ^{24}_{12}\text{Mg} + \beta^- + \bar{\nu} \]

\( 4^+ \rightarrow 4^+ \) preferred by structure (99.9%)
\( 4^+ \rightarrow 0^+ \) preferred by energetics
B. Negatron Decay

1. Mechanism

$$\text{NEGATRON} \equiv 0_{\text{\text{-}1}}^\beta^- \quad \text{(i.e., electron of nuclear origin)}$$

$$\begin{bmatrix} A \\ Z \end{bmatrix} X \rightarrow \begin{bmatrix} A \\ Z+1 \end{bmatrix} Y + 0_{\text{-1}}^\beta^- + \bar{\nu} + Q_{\beta^-}$$

[Y$^{+1}$] is initial chemical state-cation
Nuclear reactors: weird cation chemistry

2. Examples

$$\begin{align*}
^3_1\text{H} & \rightarrow ^3_2\text{He} + 0_{\text{-1}}^\beta^- + \bar{\nu} \\
^{14}_6\text{C} & \rightarrow ^{14}_7\text{N} + 0_{\text{-1}}^\beta^- + \bar{\nu}
\end{align*}$$

3. Energetics

a. $$Q_{\beta^-} = \Delta(A X) - [\Delta(A Y)^+ + \Delta e^- + \Delta v]$$

$$\Delta v \approx 0;$$
\[
\Delta(^AY) = \Delta(^AY^+) + \Delta e^-
\]

\[
Q_{\beta^-} = \Delta(^AX) - \Delta(^AY)
\]

\[
\therefore \text{ IF } \Delta(X) > \Delta(Y) ; \beta^- \text{ emission is possible }
\]

b. Observed Q-values: \[ Q_{\beta^-} \approx 5 \text{ MeV} \quad (\beta^- \text{ is relativistic}) \]

c. Spectrum

\[
P(\beta^-)
\]

Spectrum is continuous instead of discrete lines as in \( \alpha \) (and \( \gamma \)) decay. First evidence for neutrinos. If 2-body, then \( E_{\beta^-} \approx Q_{\beta^-} \)

\[
E_{\beta^-} \quad Q_{\beta^-} \text{ (end point)}
\]

For a three-body breakup, energy and momentum can be shared in an infinite number of ways;
C. Positron Decay: \( {^0_1\beta^+} \) (antiparticle of electron)

1. Mechanism:

\[
\begin{bmatrix}
\text{AX} & \rightarrow & {^0_1\beta^+} + \text{AY} + \nu
\end{bmatrix}
\]

CHARACTERISTIC OF 
PROTON-RICH NUCLEI;
OCCURS INSIDE NUCLEUS

\([Y^{-1}]\) is initial chemical state; anion

2. Example:

\[
\frac{11}{6} \text{C} \rightarrow {^0_1\beta^+} + \frac{11}{5} \text{B} + \nu + Q_{\beta^+} \quad (t_{1/2} = 21 \text{ min})
\]

Used to tag biomolecules; \(^{13}\text{N} + ^{15}\text{O}\) also, medical cyclotron

3. Energetics

a. \( Q_{\beta^+} = \Delta(X) - [\Delta(Y^-) + \Delta(\beta^+) + \Delta(\nu)] \); \( \Delta(e^-) = \Delta(\beta^+) \)

\[
Q_{\beta^+} = \Delta(X) - [\Delta(Y) + \Delta(e) + \Delta(\beta^+) + 0] \; ; \; \Delta(\nu) \approx 0
\]

\[
= \Delta(X) - \Delta(Y) - 2\Delta e \; ; \; \Delta(e^-) = 0.511 \text{ MeV}
\]

\[
Q_{\beta^+} = \Delta(X) - \Delta(Y) - 1.022 \text{ MeV}
\]
NOTE: Mass of antiparticle in matter world is equal to the mass of the particle. Creation requires twice the mass of the particle to raise it from antimatter state; energy is stolen from the rest of the nucleus.

b. Result: For $\beta^+$ decay to occur:

$$\Delta(X) - \Delta(Y) \geq 1.022 \text{ MeV}$$

c. $Q_{\beta^+} \approx 5 \text{ MeV}$

4. Relative Spectrum Shape
Coulomb field of nucleus attracts $\beta^-$, but repels $\beta^+$; shape is distorted
5. **Fate of Positron**
   
a. **Thermalization:** collisions with other electrons reduce kinetic energy to room temperature (3 kT/2)

b. **Positronium Formation:** (Lightest molecule)

   \[ e^- + e^+ \]  
   \[ t_{1/2} \sim 10^{-10} \text{ s} \]  
   \[ \text{spins} \]  
   \[ \text{ortho} \uparrow \downarrow ; J = 1 \]  
   \[ \text{para} \downarrow \uparrow ; J = 0 \]  

   Lifetime depends on chemical environment

c. **Annihilation**

   \[ e^+ + e^- \rightarrow 2\gamma \]  
   \[ E_\gamma = 0.511 \text{ MeV} = M_e c^2 \]  
   annihilation radiation (180 deg. apart)

d. **Radioactive Tag** Highly specific

   - BACK-TO-BACK \( \gamma \)s
   - Monoenergetic 0.511 MeV gamma
   - Coincidence detection
   - PET: positron emission tomography
C. Electron Capture Decay -- EC
Alternate mechanism to positron decay for proton-rich nuclei.
SAME NET RESULT for DAUGHTER

\[ ^1_1H + ^0_-1e^- \rightarrow ^0_0n + \nu + Q_{EC} \]
a. **Generic Equation**

\[
\frac{AX^+}{Z} + ^0e^- \rightarrow \frac{AY}{Z-1} + \nu + Q_{EC}
\]

However, the electrons are not in the ground state.

b. **Example:**

\[
\frac{37}{18}Ar + ^0e^- \rightarrow \frac{37}{17}Cl + \nu \quad (t_{1/2} = 35.0 \text{ d})
\]

reverse reaction of solar neutrino experiment

c. \( Q_{EC} = \Delta(X) - \Delta(Y) \)

\[ \therefore \text{If } \Delta(X) > \Delta(Y) \]

EC is possible

Note: \( Q_{EC} = Q_{\beta^+} + 1.022 \text{ MeV} \)
2. Atomic Effects

a. **Capture Process:** occurs preferentially from atomic orbitals nearest nucleus (low n) – highest probability for being inside the nucleus:

\[
\begin{align*}
1s & \\
2s & \\
3s & \\
\end{align*}
\]

K-shell    L-shell    M-shell

RESULT: Capture Probability \( P_{EC}(K) > P_{EC}(L) > P_{EC}(M) \) … etc.

b. For fixed principal quantum number \( n \), capture occurs from lowest \( \ell \) state; i.e., \( s \) state has a higher probability of being inside nucleus.

\[
P_{EC}(ns) > P_{EC}(np) > P_{EC}(nd) \text{ etc.}
\]

\[
\begin{align*}
s & \\
p & \\
d & \\
\end{align*}
\]
3. Radiation from EC
   a. Primary: monoenergetic neutrino (hard to detect)
      heavy recoil nucleus (hard to detect)

   b. Secondary: Detectable Radiation
      - x-rays – emitted during electronic orbital rearrangement
        x-rays are characteristic of DAUGHTER nucleus. WHY?
      - Auger electrons – x-rays may interact with outer orbital electrons
        and eject low energy electrons instead
        (internal photoelectric effect)

\[
\frac{P(x - \text{ray})}{P(\text{Auger})} \propto Z
\]

REASON: x-ray energy increases with Z; \( \therefore \) \( \lambda \) too short to interact.

- Gamma rays – If EC populates an excited state of daughter nucleus,
  monoenergetic \( \gamma \)-ray will be emitted.

   c. EC most difficult to detect of all decay modes
4. Chemical Effects

EC is only decay mode affected by the chemical environment

a. Ligand dependence: \( t_{1/2}(\text{Be}) < t_{1/2}(\text{BeF}_2) \): \( \text{F} \leftarrow \text{Be} \rightarrow \text{F} \)

b. Pressure dependence: High pressure shortens half-life

c. Stars: \( ^7\text{Be} \) cannot decay, no e\(^-\)s; \( Q_{\beta^+} = -0.16 \text{ MeV} \)

5. EC/\( \beta^+ \) Competition

a. If \( Q_{\text{EC}} < 1.022 \text{ MeV} \), EC ONLY
   If \( Q_{\text{EC}} \geq 1.022 \text{ MeV} \), EC & \( \beta^+ \)

b. If both EC and \( \beta^+ \) are possible:

\[
\frac{P(\text{EC})}{P(\beta^+)} \propto Z
\]

For \( A > 180 \), EC predominates;

Factors:
• Coulomb barrier inhibits \( \beta^+ \)
• Probability that \( 1s \) electron is inside the nucleus increases with increasing \( Z \) and \( A \).
IV. Gamma Decay

Analogous process to photon emission from atoms and molecules (uv, x-rays, IR …)

\[ \frac{A}{Z} X^* \rightarrow \frac{A}{Z} X \rightarrow 0_{\gamma} + \frac{A}{Z} X \]

where \( m \equiv * \) = excited state

\[ E_{\gamma} \approx Q_{\gamma} \] (recoil energy negligible)

i.e. Nucleus changes its energy state

\[ Q_{\gamma} = E_{\gamma} + E_R \ ; \ E_R \sim eV \]
A. **Occurrence**

1. De-excitation after nuclear reactions or radioactive decay.

   \[ t_{1/2} (\gamma) \gtrsim 10^{-14} \text{s} \quad \text{ELECTROMAGNETIC INTERACTION} \]

2. \[ t_{1/2} (\gamma) = f (E_\gamma, \Delta I, \Delta \pi) \]
   - **Short** \( t_{1/2} (\gamma) \): Large \( E_\gamma \), \( \Delta I = 0 \), \( \Delta \pi = \text{NO} \)
   - **Long** \( t_{1/2} (\gamma) \): Small \( E_\gamma \), \( \Delta I = \text{large} \), \( \Delta \pi = \text{YES} \)

3. **Isomers:** \[ t_{1/2} \gtrsim 10^{-6} \text{s} \quad \text{(arbitrary definition)} \]
   Unusually long-lived \( \gamma \)-ray emitters

**NOTE:** For radioactive labeling of compounds, would like

\[ t_{1/2} \sim \text{hours} – \text{days} \Rightarrow \text{stable product} \]
B. Competing Mechanisms for $\gamma$-decay

1. Photon Emission:
   Two-body decay $\rightarrow$ DISCRETE ENERGIES $E_\gamma$

2. Internal Conversion: IC

   a. Excess nuclear energy transferred to an atomic electron

   $$\text{Am}^Z_{X} \rightarrow \text{A}^{X+}_{Z} + e^-$$

   $E_e = E_\gamma - BE(e^-)$
   $BE(e^-) = \text{electron binding energy}$

   b. Distinguishing Electrons observed in Nuclear Decay

   **IC**: $E_e$ is **monoenergetic**; Discrete spectra: $E_e \sim E_\gamma$ (MeV)

   **EC Auger**: $E_e$ is **monoenergetic**; Discrete peak: $E_e \sim E_{xray}$ (eV)

   **Negatron Decay**: Spectrum **continuous**: $E_{\text{max}} \sim Q_{\beta^-}$
c. **Atomic Rearrangement**
Vacancy in inner shell (same as EC)
- IC is followed by x-rays and Auger electrons
- IC – x-rays characteristic of parent ($\Delta Z = 0$)
- EC – x-rays characteristic of daughter ($\Delta Z = -1$)

d. **Competition between $\gamma$ and IC:**
- Large $Z$ – Favors IC; more compact orbitals; higher probability that $e^-$ is inside nucleus
- Large $\Delta I$ – favors IC; since $e^-$ has mass, it can carry away angular momentum easier.
- Low $E_\gamma$ – favors IC; wavelength of $\gamma$ close to that of atomic electrons; high $E_\gamma \Rightarrow$ short wavelength
2.3. GAMMA DECAY

\[ E_{1c} \text{ electron} \rightarrow E_{\text{trans}} - B.E. \text{ atomic electron} \]

\[ \text{nucleus} \rightarrow e^- \rightarrow \text{Auger electron} \]

\[ e^- \rightarrow \text{nucleus} \rightarrow \text{K, L, M electron shells} \]

\[ ^{113}\text{In} \rightarrow ^{113}\text{In} (1c, 392 \text{ keV}) \]

\[ \text{Number of IC electrons} \]

\[ E_{\text{electron}} \rightarrow \text{K, 360 keV} \]

\[ L, 385 \text{ keV} \]

\[ M, 389 \text{ keV} \]

\[ B.E. K = 32 \text{ keV} \]

\[ B.E. L = 7 \text{ keV} \]

\[ B.E. M = 3 \text{ keV} \]

Figure 2.7. Internal conversion decay.

\[ \text{Ehmann + Vance} \]
3. Pair Production

\[ ^{\text{Am}}_{\text{Z}}X \rightarrow ^{\text{A}}_{\text{Z}}X + \beta^- + \beta^- + Q_{\text{pp}} \]

REVERSE OF ANNIHILATION

Electron-positron pair production. A high-energy gamma ray coming in from above scatters off an atomic electron, losing some of its energy and producing an energetic recoil electron and an electron-positron pair. The electron and positron paths curve because the chamber is placed in a strong magnetic field. The direction of the curves reveals the signs of the particles' charges.
\[ Q_{pp} = \Delta(A^m X) - \left[ \Delta(A X) + \Delta(\beta^+) + \Delta(\beta^-) \right] \]

\[ \Delta(\beta^+) = \Delta(\beta^-) = \Delta(e^-) \]

\[ \Delta(A^m X) - \Delta(A X) = Q_{\gamma} \]

\[ Q_{pp} = Q_{\gamma} - 1.022 \text{MeV} \]

::: Pair Production cannot occur unless \( Q_{\gamma} \geq 1.022 \text{ MeV} \)

b. Momentum Conservation: \( p(\beta^+) \Rightarrow E_{\beta^+} = E_{\beta^-} \)

\[ E_{\beta^\pm} = \frac{E_{\gamma} - 1.022 \text{MeV}}{2} \]

NOTE: eventually the \( \beta^+ \) annihilates and gives 1.022 MeV back
4. All Three Modes Compete

<table>
<thead>
<tr>
<th>Mode</th>
<th>Relevant Energy</th>
<th>&quot;Observe&quot;</th>
<th>Observed Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low $E_\gamma$</td>
<td>$\gamma \rightarrow IC$</td>
<td>($\lesssim 0.2 \text{ MeV}$)</td>
<td>ELECTRON</td>
</tr>
<tr>
<td>Medium $E_\gamma$</td>
<td>$\gamma \rightarrow \gamma$</td>
<td>($\sim 1 \text{ MeV}$)</td>
<td>PHOTON</td>
</tr>
<tr>
<td>High $E_\gamma$</td>
<td>$\gamma \rightarrow \beta^\pm_{\text{pair}}$</td>
<td>($\gtrsim 2 \text{ MeV}$)</td>
<td>$\beta^- - \beta^+ \text{ pair}$</td>
</tr>
</tbody>
</table>