C. Alpha Decay Probability

1. Energetics: Q_α positive for all A>140 nuclei

2. Range of Measured Half-Lives (~10^{44})

   10^{16} y > $t_{1/2}$ > 10^{-21} s

3. Why α ?

a. Proton & Neutron Emission: Q_p , Q_n are negative near valley of beta stability (peak of peninsula); Thermodynamically forbidden

b. Other Nuclei ; e.g. ^{12}C, ^{16}O …

- $Q(^{12}C)$, $Q(^{16}O)$ positive ; therefore possible
- Probability is low (i.e., $t_{1/2}$) is long) -- $P(^{14}C)/P\alpha \sim P^{-10}$ (Exotic decay mode)

$$^{232}_{90}Th \rightarrow {}^{14}_{6}C + {}^{218}_{84}Po$$

FACTS

- 11.7 MeV α particles from ^{212m} Po are the highest energy alphas from a radioactive source
- 2.0 MeV alphas from Sm are the among lowest energy alphas from a radioactive source
- Most alpha particles from radioactive sources fall in the range of 4-8 MeV.
- **Associated with this narrow range in energy is the enormous range in half-life noted above**.

WHY?

4. Coulomb Barrier Penetration

What is the relevant potential for the nucleus?
We can consider this by examining the approach of an alpha particle from infinity (microscopic reversibility).
If the collision is head-on (dead center) there are two parts to consider the attractive nuclear potential and the repulsive Coulomb potential.

$V(r) = V_{nuclear} + V_{Coulomb}$

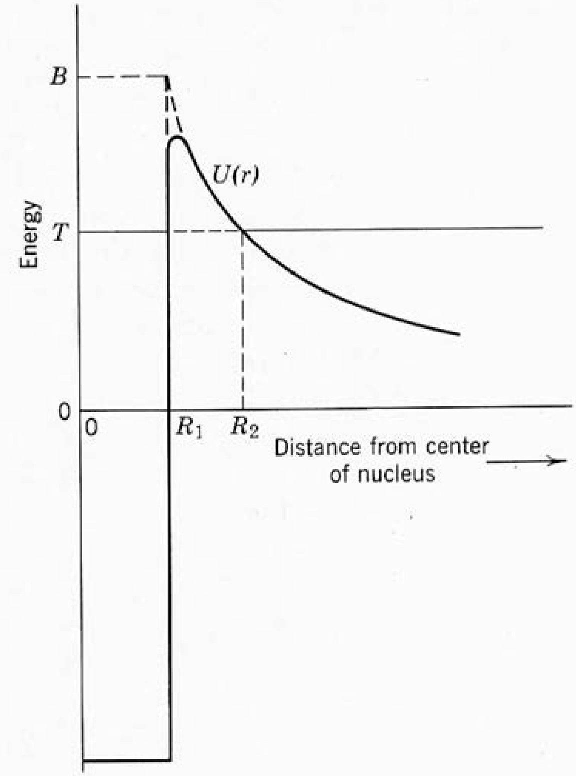
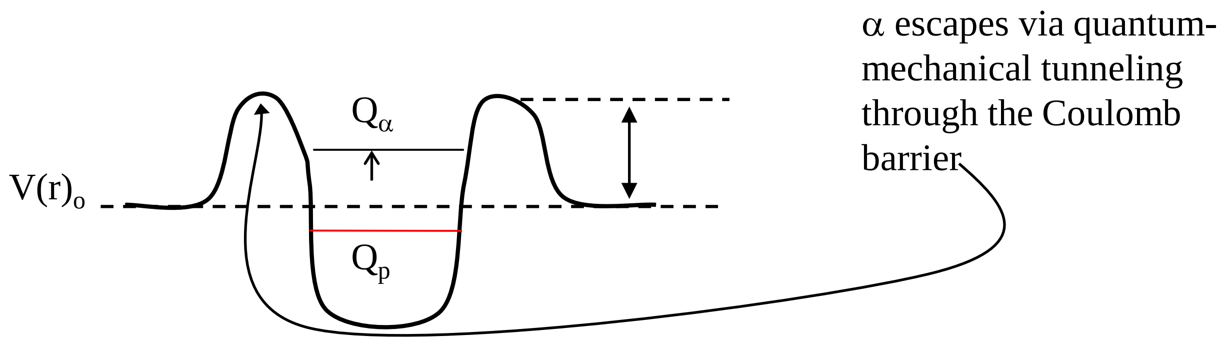


Fig. 8-2 Potential energy in the neighborhood of a nucleus.



a. Electrostatic Repulsion Energy for Two Charged Spheres:

$$V_{Coulomb} = \frac{(Z_1 e)(Z_2 e)}{R} = 1.44 \frac{Z_1 Z_2}{R} MeV.fm$$

where
$Z_1 = 2$
$Z_2 = Z_{recoil}$
$R = R_1 + R_2 = r_0 \, (A_1^{1/3} + 4^{1/3})$

b. Example: $^{220}_{102}\text{No}$ let $r_0 = 1.4$ fm

$$V_{Coulomb} = \frac{1.44(2)(100)}{1.4(4^{1/3} + 216^{1/3})} = 25.5 MeV$$

Typical of most heavy nuclei

c. $Q_\alpha \lesssim 10$ MeV ; $\therefore$ $V_{coul} >> Q_\alpha$ and can't escape classically

d. Relative Barriers (approximate)

$V_{coul}(\alpha)$: $V_{coul}(^{12}C)$: $V_{coul}(^{16}O) \approx 2:6:8$ (since Z & R of recoil ≈ constant)
$\therefore$ heavier fragments have much higher (and thicker) hill to punch through

5. Probability – P_α

a. Tunneling Probability:

$$P_\alpha \propto (P_{formation})e^{(Q_\alpha - V_{Coulomb})}$$

$$P_\alpha \propto \frac{1}{t_{1/2}(\alpha)}$$

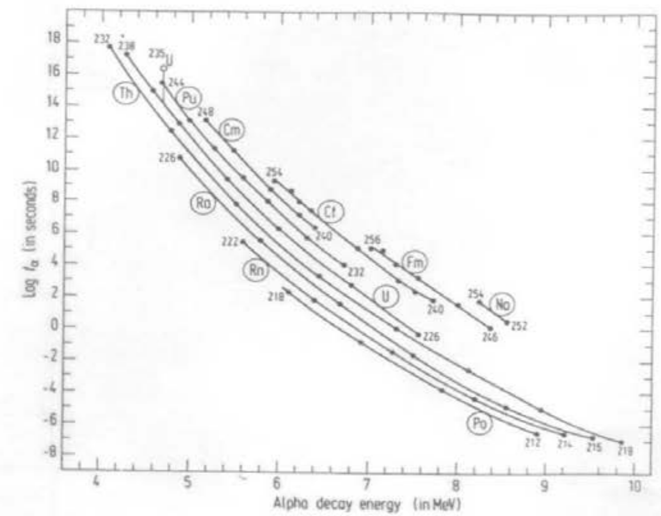
b. P_α favored by:

(1) large Q_α and low V_{coul} or small $Q_\alpha - V_c$

$\therefore t_{1/2}(\alpha) \propto 1/P(\alpha) \propto e^{-(Q_\alpha - V_c)}$

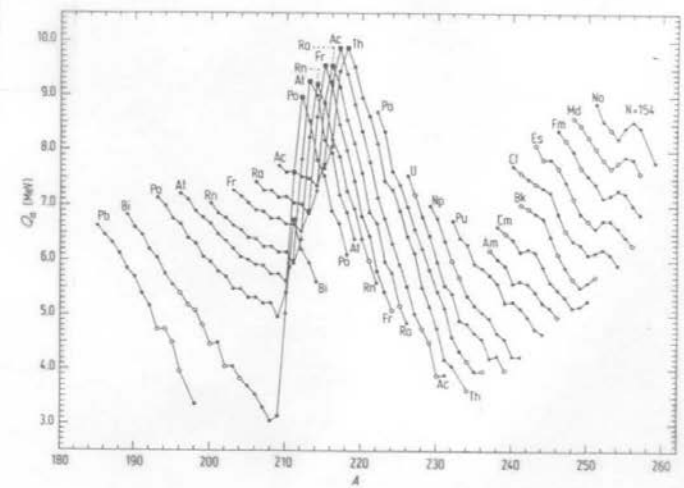
or $\log t_{1/2} \propto V_c - Q_\alpha$

Geiger-Nuttal law



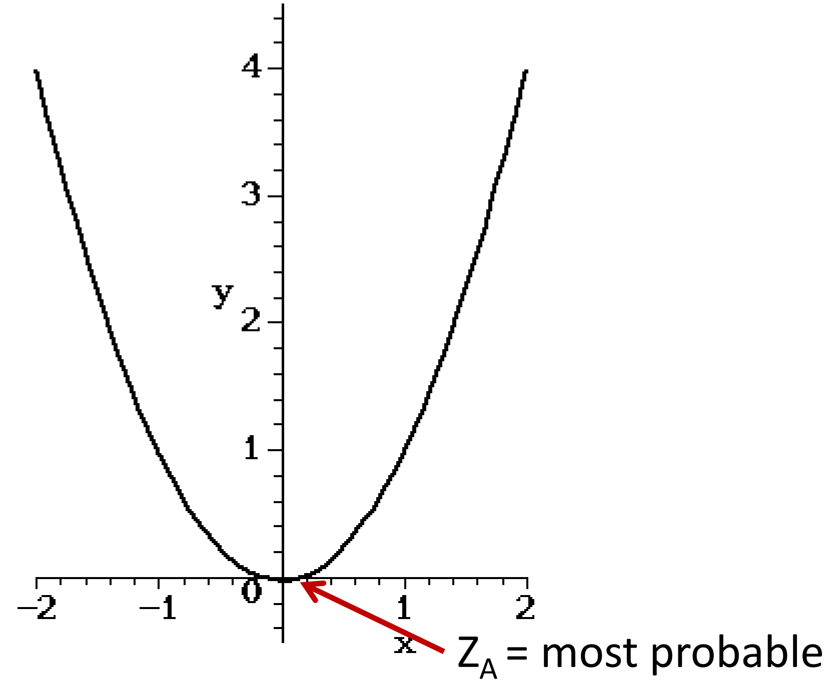
D. Applications/Environment

1. ^{241}Am (458y) – smoke detectors
2. ^{238}Pu (88y) – remote sensing devices; power sources
3. ^{222}Rn (3.8d) – natural radioactivity – health effects
4. ^{226}Ra (1620y) – cancer therapy
5. (^{210}Po) (138d) – power sources
6. Dating tags – U,Th, Sm



Beta-Decay: Neutron-Proton Transformations

Connects nuclei that are isobars



NEUTRON EXCESS

Negatron Decay

$$n \rightarrow {}^{1}H + \beta^{-} + \overline{\nu}$$

PROTON EXCESS

Positron/electron capture Decay

$${}^{1}H \rightarrow {}^{1}n + \beta^{+} + \nu$$

$${}^{1}H + e^{-} \rightarrow {}^{1}n + \nu$$

A. **Beta-decay Half-lives**

1. Factors that Influence Rate

a. <u>WEAK NUCLEAR FORCE</u>:

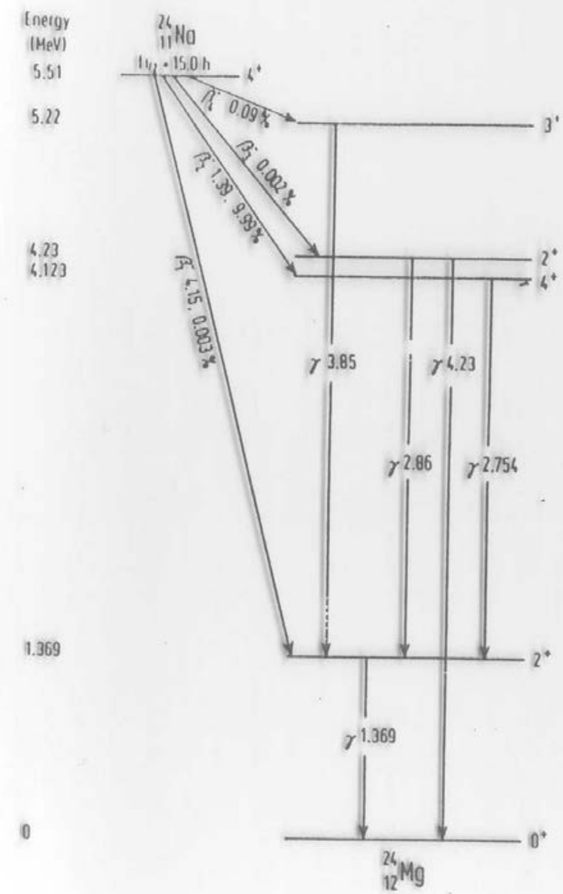
$\therefore$ SLOW !!! $t_{1/2}(\beta^{\pm}) \geq 10^{-3}$ s (cf $\gtrsim 10^{-21}$ s for α)

b. <u>Large Q_{β^-}</u> $\Rightarrow$ Short half-life

c. Structure: spin and parity changes (ΔI & $\Delta\pi$) retard rates

2. Example

$$^{24}_{11}Na \rightarrow {}^{24}_{12}Mg + \beta^- + \bar{\nu}$$



$4^+ \rightarrow 4^+$ preferred by structure (99.9%)

$4^+ \rightarrow 0^+$ preferred by energetics

B. Negatron Decay

1. Mechanism

NEGATRON $\equiv {}_{-1}^{0}\beta^{-}$ (i.e., electron of nuclear origin)

$$\left[{}_{Z}^{A}X \dashrightarrow {}_{Z+1}^{A}Y + {}_{-1}^{0}\beta^{-} + \bar{\nu} + Q_{\beta^{-}} \right]$$

CHARACTERISTIC OF NEUTRON-EXCESS NUCLEI

[Y^{+1}] is initial chemical state-cation

Nuclear reactors: weird cation chemistry

2. Examples

$${}_{1}^{3}H \rightarrow {}_{2}^{3}He + {}_{-1}^{0}\beta^{-} + \bar{\nu}$$

$${}_{6}^{14}C \rightarrow {}_{7}^{14}N + {}_{-1}^{0}\beta + \bar{\nu}$$

3. Energetics

a. $Q_{\beta^-} = \Delta(^{A}X) - [\Delta(^{A}Y)^{+} + \Delta_{e^-} + \Delta_{\nu}]$

$\Delta_{\nu} \approx 0$;

$\Delta(^{A}Y) = \Delta(^{A}Y^{+}) + \Delta_{e^-}$

$Q_{\beta^-} = \quad \Delta(^{A}X) - \Delta(^{A}Y)$

$\therefore$ IF $\Delta(X) > \Delta(Y)$; β^- emission is possible

b. Observed Q-values: $Q_{\beta^-} \lesssim 5$ MeV (β^- is relativistic)

c. Spectrum

$P(\beta^-)$

E_{β^-} $\qquad Q_{\beta^-}$ (end point)

Spectrum is continuous instead of discrete lines as in α (and γ) decay. First evidence for neutrinos. If 2-body, then $E_{\beta^-} \approx Q_{\beta^-}$

For a three-body breakup, energy and momentum can be shared in an infinite number of ways;

C. Positron Decay: $^{0}_{1}\beta^{+}$ (antiparticle of electron)

1. Mechanism:

$$\left[{}^{A}_{Z}X \dashrightarrow {}^{0}_{1}\beta^{+} + {}^{A}_{Z-1}Y + \nu \right]$$

CHARACTERISTIC OF PROTON-RICH NUCLEI; OCCURS INSIDE NUCLEUS

$[Y^{-1}]$ is initial chemical state ; anion

2. Example:

$${}^{11}_{6}C \rightarrow {}^{0}_{1}\beta^{+} + {}^{11}_{5}B + \nu + Q_{\beta^{+}} \qquad (t_{1/2} = 21 \text{ min})$$

Used to tag biomolecules; ^{13}N + ^{15}O also, medical cyclotron

3. Energetics

a. $Q_{\beta}+ = \Delta(X) - [\Delta(Y^{-}) + \Delta(\beta^{+}) + \Delta(\nu)]$; $\Delta(e^{-}) = \Delta(\beta^{+})$

$Q_{\beta}+ = \Delta(X) - [\Delta(Y) + \Delta(e) + \Delta\beta^{+} + 0]$; $\Delta(\nu) \approx 0$

$= \Delta(X) - \Delta(Y) - 2\Delta e$; $\Delta(e^{-}) = 0.511$ MeV

$$Q_{\beta}+ = \Delta(X) - \Delta(Y) - 1.022 \text{ MeV}$$

NOTE: Mass of antiparticle in matter world is equal to the mass of the particle. Creation requires twice the mass of the particle to raise it from antimatter state; energy is stolen from the rest of the nucleus.

b. Result: For β^+ decay to occur:

$$\Delta(X) - \Delta(Y) \geq 1.022 \text{ MeV}$$

c. $Q_{\beta}+ \lesssim 5$ MeV

4. Relative Spectrum Shape

Coulomb field of nucleus attracts β-, but repels β+ ; shape is distorted

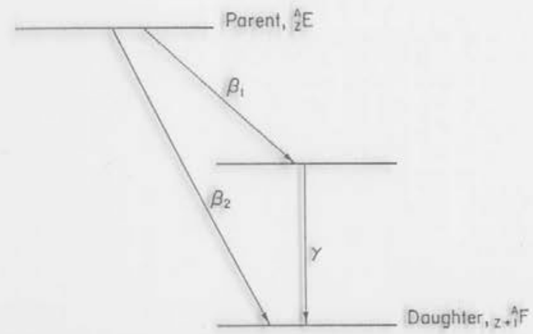
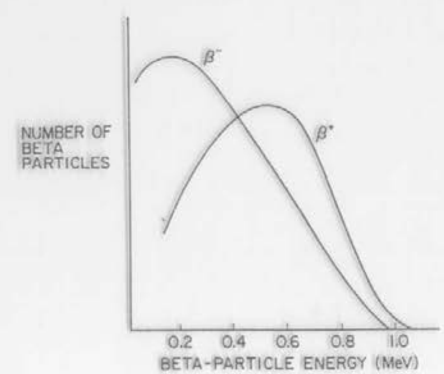


Figure 2.4. Decay scheme for a radionuclide undergoing negatron decay. Two β decay paths are shown. A γ ray is emitted from an excited state of the daughter in one decay path.

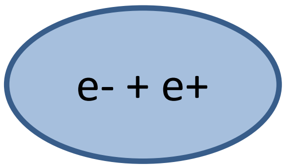
TYPES OF RADIOACTIVE DECAY



5. Fate of Positron

a. Thermalization: collisions with other electrons reduce kinetic energy to room temperature (3 kT/2)

b. Positronium Formation: (Lightest molecule)



$t_{1/2} \sim 10^{-10}$ s

spins

ortho ↑↑ ; J = 1

para ↓↑ ; J = 0

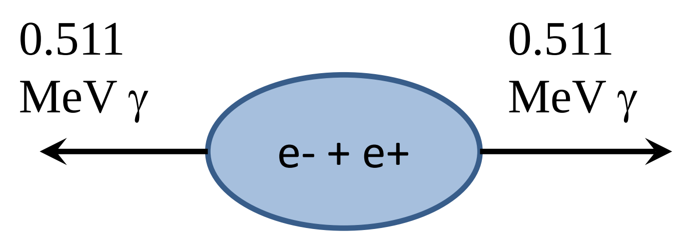
Lifetime depends on chemical environment

c. Annihilation

$e^+ + e^- \rightarrow 2\gamma$; $E_\gamma = 0.511$ MeV $= M_e c^2$

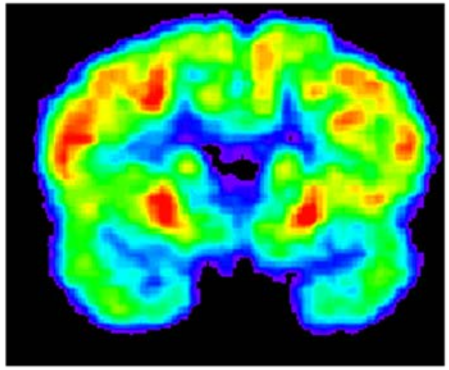
annihilation radiation (180 deg. apart)

d. Radioactive Tag Highly specific



- BACK-TO-BACK γs
- Monoenergetic 0.511 MeV gamma
- Coincidence detection
- PET: positron emission tomography

a)



b)

C. Electron Capture Decay -- EC

Alternate mechanism to positron decay for proton-rich nuclei.

SAME NET RESULT for DAUGHTER

$$\underbrace{{}^{1}_{1}H + {}^{0}_{-1}e^{-}}_{{}^{1}_{1}H^{0}} \rightarrow {}^{1}_{0}n + \nu + Q_{EC}$$

a. Generic Equation

$$ {}^{A}_{Z}X^{+} + {}^{0}_{-1}e^{-} \rightarrow {}^{A}_{Z-1}Y + \nu + Q_{EC} $$

$$ {}^{A}_{Z}X \leftarrow \text{neutral} $$

However, the electrons are not in the ground state.

b. Example:

$$ {}^{37}_{18}Ar + {}^{0}_{-1}e^{-} \rightarrow {}^{37}_{17}Cl + \nu \qquad (t_{1/2} = 35.0\ d) $$

reverse reaction of solar neutrino experiment

c. $Q_{EC} = \Delta(X) - \Delta(Y)$

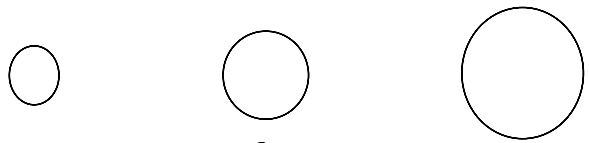
$\therefore$ If $\Delta(X) > \Delta(Y)$

EC is possible

Note: $Q_{EC} = Q_{\beta^+} + 1.022$ MeV

2. Atomic Effects

a. <u>Capture Process:</u> occurs preferentially from atomic orbitals nearest nucleus (low n) – highest probability for being inside the nucleus:



Etc.

1s 2s 3s

K-shell L-shell M-shell

RESULT: Capture Probability $P_{EC}(K) > P_{EC}(L) > P_{EC}(M) \ldots$ etc.

b. For fixed principal quantum number n , capture occurs from lowest ℓ state ; i.e., s state has a higher probability of being inside nucleus.

$$P_{EC}(ns) > P_{EC}(np) > P_{EC}(nd) \text{ etc.}$$

s p d

3. Radiation from EC

a. <u>Primary:</u> monoenergetic neutrino (hard to detect)
heavy recoil nucleus (hard to detect)

b. <u>Secondary: </u>Detectable Radiation

- x-rays – emitted during electronic orbital rearrangement
  x-rays are characteristic of DAUGHTER nucleus. **<u>WHY?</u>**
- Auger electrons – x-rays may interact with outer orbital electrons
  and eject low energy electrons instead
  (internal photoelectric effect)

$$\frac{P(x-\text{ray})}{P(\text{Auger})} \, \alpha \, Z$$

REASON: x-ray energy increases with Z; ∴ λ too short to interact.

- Gamma rays – If EC populates an excited state of daughter nucleus, monoenergetic γ-ray will be emitted.

c. <u>EC most difficult to detect of all decay modes</u>

4. Chemical Effects

EC is only decay mode affected by the chemical environment

a. Ligand dependence: $t_{1/2}(Be) < t_{1/2}(BeF_2)$: F← Be → F

b. Pressure dependence: High pressure shortens half-life

c. Stars: ${}^{7}_{4}Be$ cannot decay, no e⁻s ; $Q_{\beta+} = -0.16$ MeV

5. EC/β^+ Competition

a. If $Q_{EC} < 1.022$ MeV, EC ONLY

If $Q_{EC} \geq 1.022$ MeV, EC & β^+

b. If both EC and β+ are possible:

$$\frac{P(EC)}{P(\beta^+)} \alpha Z$$

For A > 180, EC predominates ;

Factors:

- Coulomb barrier inhibits β+
- Probability that 1s electron is inside the nucleus increases with increasing Z and A.

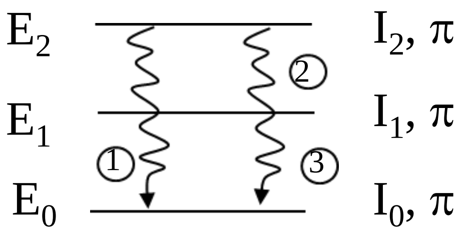
IV. Gamma Decay

Analogous process to photon emission from atoms and molecules (uv, x-rays, IR …)

$${}^{A}_{Z}X^{*} \equiv {}^{Am}_{Z}X \rightarrow {}^{0}_{0}\gamma + {}^{A}_{Z}X \qquad \text{where } m \equiv * = \text{excited state}$$

$E_\gamma \approx Q_\gamma$ (recoil energy negligible)

i.e. Nucleus changes its energy state



E_γ ① $= E_2 - E_0$

E_γ ② $= E_2 - E1$

E_γ ③ $= E_1 - E_0$

$Q_\gamma = E_\gamma + E_R$; $E_R \sim eV$

A. **Occurrence**

1. De-excitation after nuclear reactions or radioactive decay.

$t_{1/2}(\gamma) \gtrsim 10^{-14}$s   <u>ELECTROMAGNETIC INTERACTION</u>

2. $t_{1/2}(\gamma) = f\ (E_\gamma, \Delta I, \Delta\pi)$

Short $t_{1/2}(\gamma)$: Large $E_{\gamma,}$ $\Delta I = 0$, $\Delta\pi$ = NO

Long $t_{1/2}(\gamma)$: Small $E_{\gamma,}$ ΔI = large, $\Delta\pi$ = YES

3. Isomers: $t_{1/2} \gtrsim 10^{-6}$s (arbitrary definition)

Unusually long-lived γ-ray emitters

NOTE: For radioactive labeling of compounds, would like

$t_{1/2}$ ~ hours – days $\Rightarrow$ stable product

B. Competing Mechanisms for γ-decay

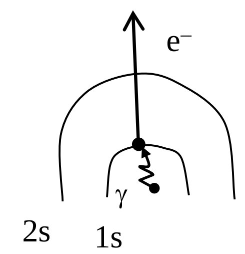
1. Photon Emission:

Two-body decay → DISCRETE ENERGIES E_γ

2. Internal Conversion: IC

a. Excess nuclear energy transferred to an atomic electron

$$^{Am}_{Z}X \rightarrow {}^{A}_{Z}X^{+} + e^{-}$$



$E_e = E_\gamma - BE(e^-)$
$BE(e^-)$ = electron binding energy

b. Distinguishing Electrons observed in Nuclear Decay

IC: E_{e^-} is monoenergetic ; Discrete spectra: $E_{e^-} \sim E_\gamma$ (MeV)

EC Auger: E_{e^-} is monoenergetic ; Discrete peak: $E_{e^-} \sim E_{xray}$ (eV)

Negatron Decay: ; Spectrum continuous: $E_{max} \sim Q_{\beta^-}$

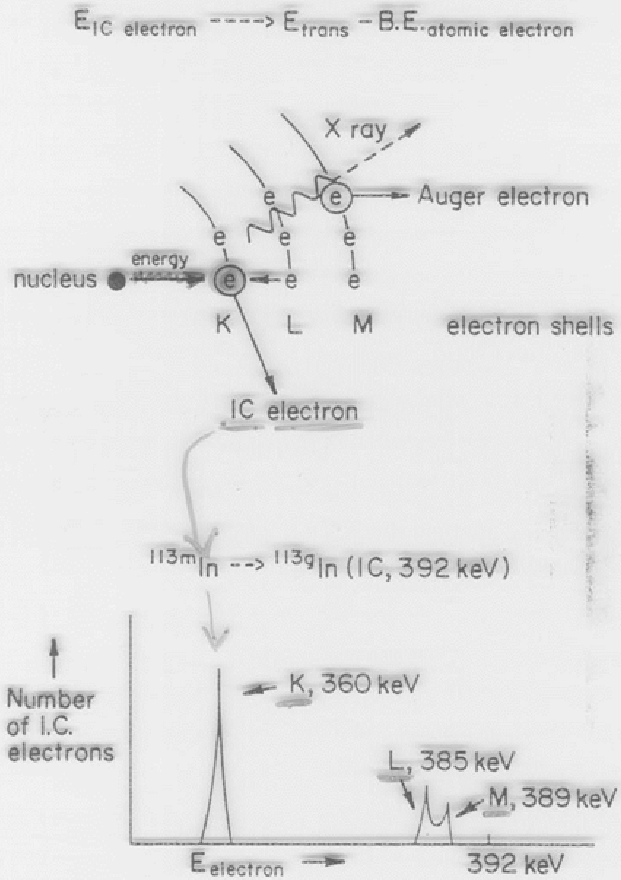
c. Atomic Rearrangement

Vacancy in inner shell (same as EC)

- IC is followed by x-rays and Auger electrons
- IC – x-rays characteristic of parent ($\Delta Z = 0$)
- EC – x-rays characteristic of daughter ($\Delta Z = -1$)

d. Competition between γ and IC:

- Large Z — Favors IC; more compact orbitals; higher probability that e^- is inside nucleus
- Large ΔI – favors IC; since e^- has mass, it can carry away angular momentum easier.
- Low E_γ – favors IC; wavelength of γ close to that of atomic electrons; high $E_\gamma \Rightarrow$ short wavelength



$BE_K = 32$ keV
$BE_L = 7$ keV
$BE_M = 3$ keV

Figure 2.7. Internal conversion decay.

↑ Ehmann + Vance

3. Pair Production

$^{Am}_{Z}X \rightarrow {}^{A}_{Z}X + \beta^{-} + \beta^{-} + Q_{pp}$

REVERSE OF ANNIHILATION

Electron-positron pair production. A high-energy gamma ray coming in **from above scatters off an atomic** electron, losing some of its energy **and producing an energetic recoil electron and an** electron-positron **pair**. The electron **and positron paths** curve **because the chamber is placed in a strong magnetic** field. The direction of **the curves reveals** the **signs of the particles' charges.**

∴

$$Q_{pp} = \Delta(^{Am}X) - \left[\Delta(^{A}X) + \Delta(\beta^{+}) + \Delta(\beta^{-})\right]$$

$\Delta(\beta^{+}) = \Delta(\beta^{-}) = \Delta(e^{-})$

$$\Delta(^{Am}X) - \Delta(^{A}X) = Q_{\gamma}$$

$$Q_{pp} = Q_{\gamma} - 1.022 MeV$$

∴ Pair Production cannot occur unless $Q_{\gamma} \geq 1.022$ MeV

b. <u>Momentum Conservation</u>: p(β^{+}) $\Rightarrow$ $E_{\beta^{+}} = E_{\beta^{-}}$

$$E_{\beta\pm} = \frac{E_{\gamma} - 1.022 MeV}{2}$$

NOTE: eventually the β^{+} annihilates and gives 1.022 MeV back

4. All Three Modes Compete

		mode	relevant energy	"observe"	observed energy
Low E_γ	$\rightarrow$	IC	($\lesssim$ 0.2 MeV)	ELECTRON	$Q_\gamma - E_{BE}$
Medium E_γ	$\rightarrow$	γ	(~ 1 MeV)	PHOTON	Q_γ
High E_γ	$\rightarrow$	$\beta^{\pm}_{pair}$	($\gtrsim$ 2 MeV)	$\beta^- - \beta^+$ pair	$Q_\gamma - 1.022$