Nuclear Shell model

C. Prediction of spins and Parities: GROUND RULES

1. Even-Even Nuclei

$I \pi = 0 +$

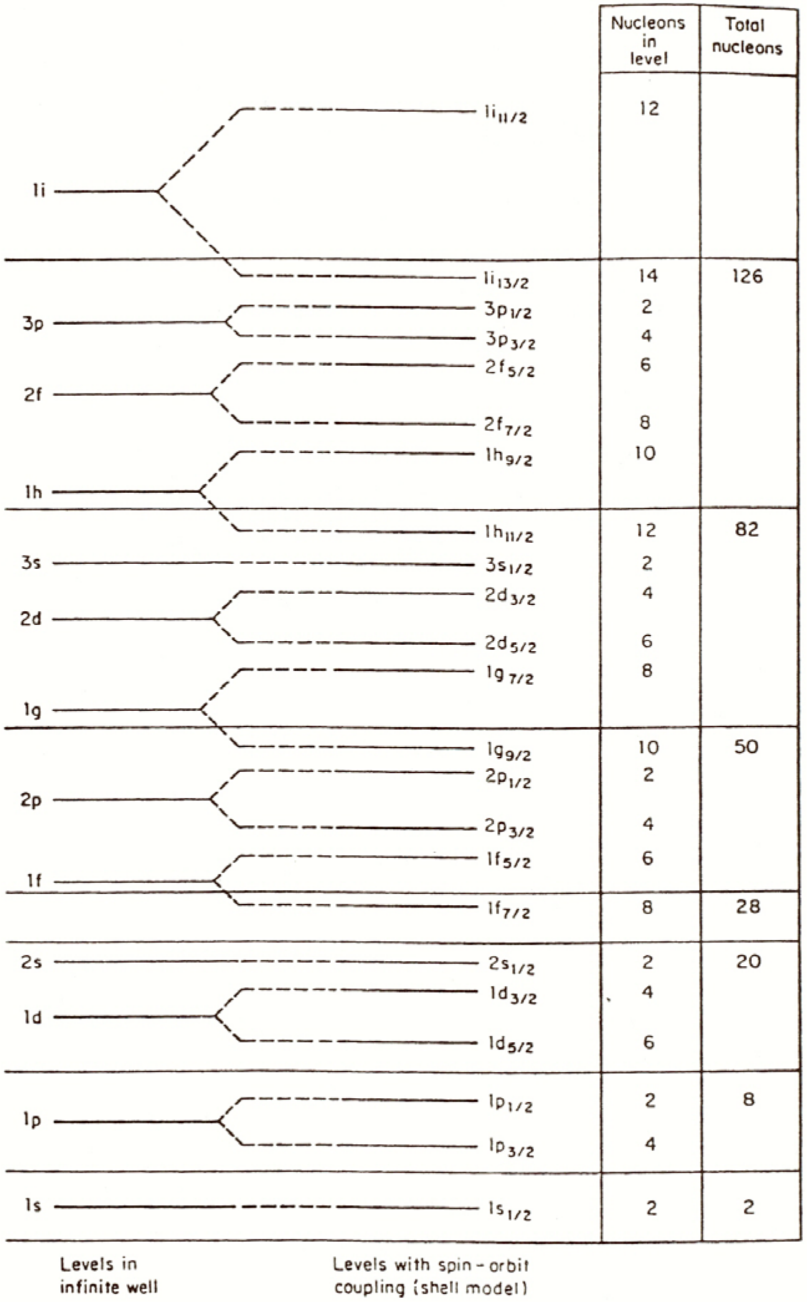
RULE: All nucleon orbitals are filled pairwise,
i.e., ν, l, j, m_j state followed by $\nu, l, j, -m_j$ state

NO EXCEPTIONS

2. Odd-A Nuclei

INDEPENDENT PARTICLE ASSUMPTION
Nucleons fill orbitals pairwise up to last odd nucleon.
RULE: Last odd nucleon determines quantum properties of entire nucleus

Result:
a. ${}^{A-1}X$ core is e-e; ∴ 0+
b. Last particle Iπ given by HO model with strong spin-orbit coupling;
c. Total Nucleus
$I = (\text{core}) + (\text{last nucleon}) = 0 + j = j$
$\pi = \pi\,(\text{core}) \times \pi\,(\text{last nucleon}) = + \bullet \pm = \pm$



Comparison of levels in an infinite potential well with spin-orbit coupling level sequence

NOTE: On figure of energy levels with spin-orbit coupling, parity alternates from shell to shell ($\nu \rightarrow \nu + 1$)

Filling levels: same as doing electron configurations in Bohr atom

3. Odd-Odd Nuclei

Must couple last odd proton to last odd neutron.

I = $\vec{j}_n + \vec{j}_p$

NOT COVERED: difficult angular momentum (vector additions).

4. Examples:

a. {^{12}C, ^{24}O, ^{184}Pb, ^{298}114} All Iπ = 0+

b. $^{119}_{49}In = ^{118}_{48}Cd + p$

Get from figure of energy levels with spin-orbit coupling

0^+

49th proton in level is:

1 $g_{9/2}$; j = 9/2 ; g state: π = +

$\ell = 4$

Predict Iπ = 9/2 +

This is observed.

c. : $ {}^{47}_{20}Ca : {}^{46}_{20}Ca + n $

Get from figure of energy levels with spin-orbit coupling

0^{+}

27th neutron is 1 $f_{7/2}$

∴ j = 7/2, l = 3, π = −

Predict: Iπ = 7/2−

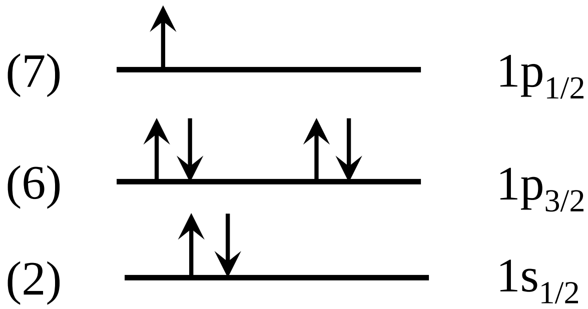
5. Bottom Line: Same counting game as in atoms ($1s^2 2s^2 2p^6 3s^2$ ...)
Works near closed shells ; deviations away from them.

D. Excited States

1. Particles and Relative Energies

Given by level scheme:

e.g. $^{15}_{8}O_{7}$ Ground state

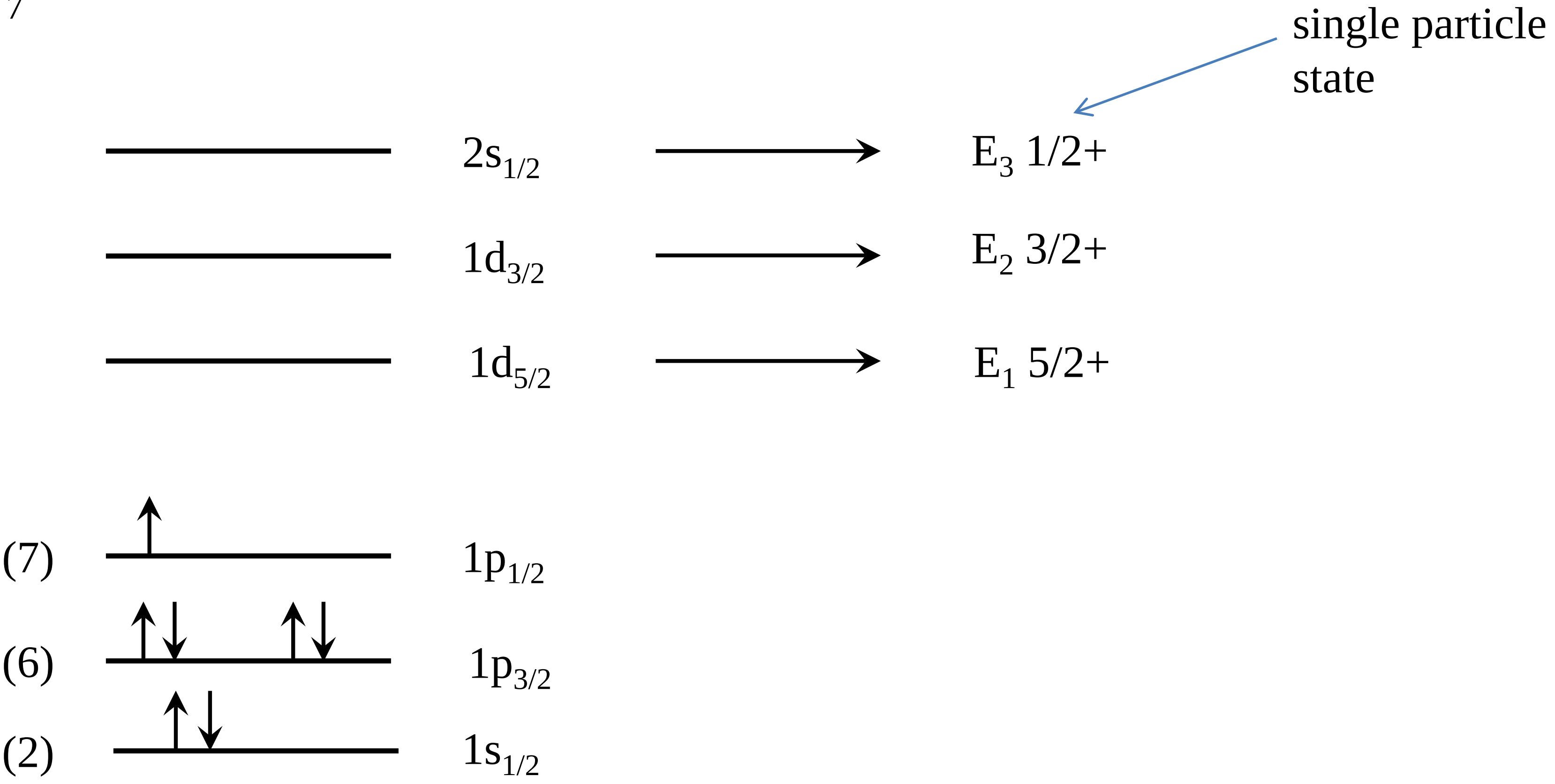


(7) $1p_{1/2}$

(6) $1p_{3/2}$

(2) $1s_{1/2}$

$^{15}_{8}O_7$ Excited state



single particle state

$2s_{1/2}$ → E_3 $1/2+$

$1d_{3/2}$ → E_2 $3/2+$

$1d_{5/2}$ → E_1 $5/2+$

(7) $1p_{1/2}$

(6) $1p_{3/2}$

(2) $1s_{1/2}$

2. Rotational and Vibrational States also exist
Due to collective motion of nucleus, superimposed on single-particle state.

E. The Shell Model and the Real World

1. Closed Shells Correct
2. Spins, Parities and Magnetic Moments – described systematically

a. e-e: Always right

b. o-A: usually correct for spherical nuclei (near closed shells). Less accurate in between

c. o-o: difficult – horseshoes

3. Low-lying energy levels – also correct near closed shells.

VIII. Unified Model

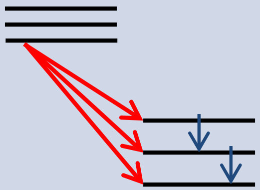
Combines LD and Shell models; allows for deformed shapes – changes order of levels between shells, but not magic numbers.

$$V(r) = V_0\left(1 - \left[\frac{ax^2}{R^2} + \frac{by^2}{R^2} + \frac{cz^2}{R^2}\right]\right)$$

Nuclear Potentials and Radioactive Decay

I. Nuclear Stability and Basic Decay Modes

A. Schematic Representation:

Synthesis		Equilibration		Decay
X+Y + Energy		$^{A}_{Z}Z^{*}$		
$\tau \leq 10^{-20}$s		$\tau \sim 10^{-16}$ - 10^{-20}s		
		Composite nucleus (Activated Complex)		

B. Stable Nuclei

1. N/Z composition: Does not change with time ⇒ peak of <BE> curve
   Kinetic vs. Thermodynamic stability; detection limit $\lesssim 10^{20}$ y

2. Total: 266
   At least one stable nucleus for all Z=1–83 EXCEPT $_{43}\mathrm{Tc}$ and $_{61}\mathrm{Pm}$

C. Radioactive Nuclei

1. Definition: A nucleus that SPONTANEOUSLY alters its neutron/proton composition or energy state ⇒ FIRST-ORDER RATE PROCESS

RADIOACTIVE DECAY IS IDENTICAL WITH AN ELEMENTARY UNIMOLECULAR DISSOCIATION IN CHEMISTRY. (A ➔ B + C)

Contrast with: nuclear reactions – n/p changes induced by collisions, 2nd order

NUCLEAR REACTIONS HAVE THE SAME FORM AS AN ELEMENTARY BIMOLECULAR CHEMICAL REACTION (A + B ➔ C + D)

2. Half-life: $t_{1/2}$

Definition: The length of time required for one-half the nuclei in a sample to disintegrate (decay):

$$N(t) = N_0 e^{-\lambda t}$$

$$\lambda = \frac{0.693}{t_{1/2}}$$

3. Primary Decay Modes

a. Alpha Decay: ${}^{4}_{2}He$ emission

b. Beta Decay: neutron ⇔ proton conversion
β specifies nuclear origin
e specifies atomic origin

c. Gamma Decay: γ , photon emission
γ = nuclear origin ; x-ray, uv, visible, ir = atomic/molecular origin

d. Exotic decay modes: fission, protons, neutrons, ^{14}C, etc.

4. Radioactivity in Nature ($t_{1/2} \gtrsim 10^8$y)

a. U–Th Decay series

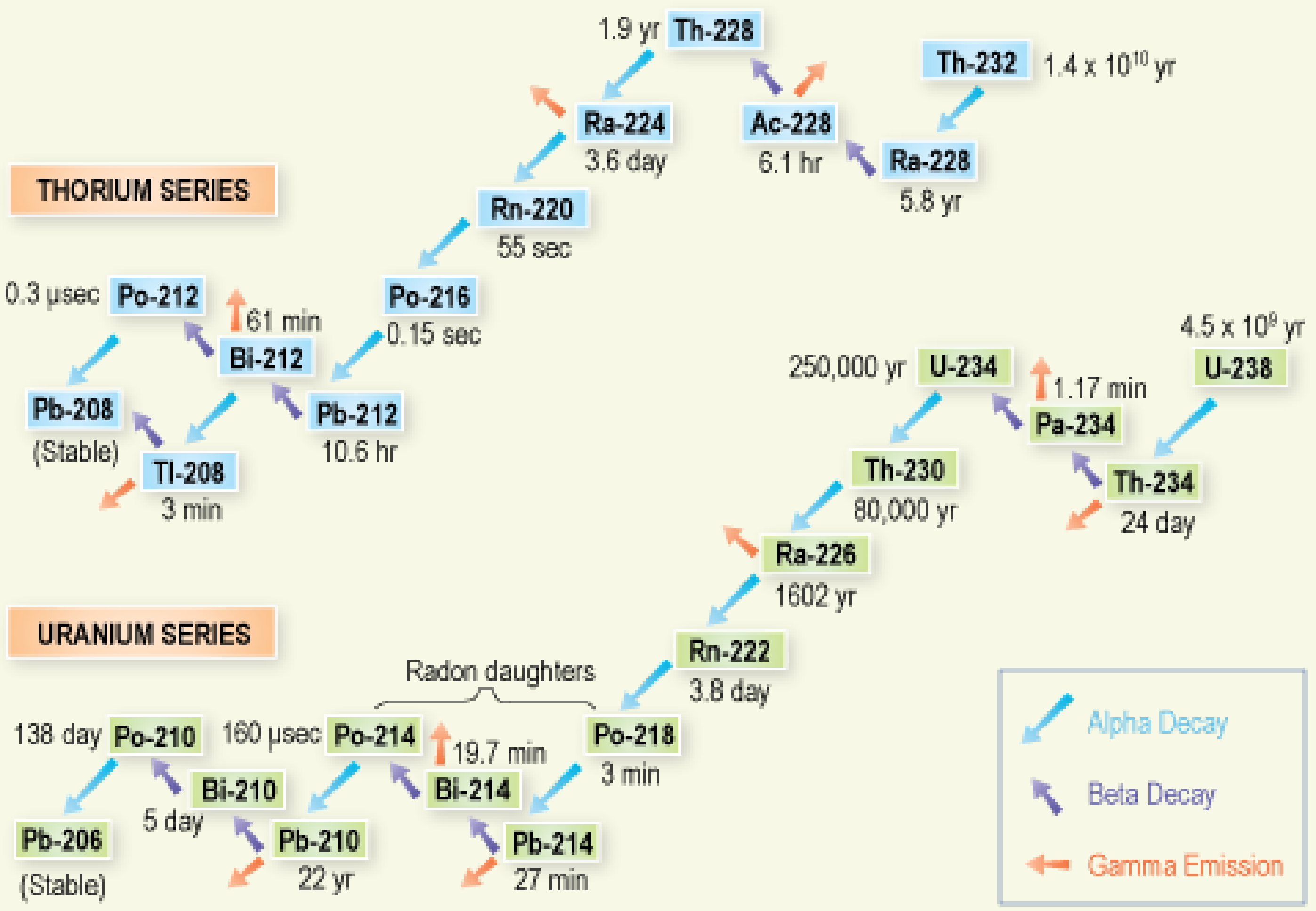
${}^{238}_{92}\mathrm{U}$	$(4.5 \times 10^9$y)	$\xrightarrow[6\beta^-]{8\alpha}$	${}^{206}_{82}\mathrm{Pb}$	(24.1%)	$A = 4n + 2$
${}^{235}_{92}\mathrm{U}$	$(7.1 \times 10^8$y)	$\xrightarrow[4\beta^-]{7\alpha}$	${}^{207}_{82}\mathrm{Pb}$	(22.1%)	$A = 4n + 3$
${}^{232}_{90}\mathrm{Th}$	$(1.4 \times 10^9$y)	$\xrightarrow[4\beta^-]{6\alpha}$	${}^{208}_{82}\mathrm{Pb}$	(52.3%)	$A = 4n$
${}^{237}_{93}\mathrm{Np}$	$(2 \times 10^6$y)	$\xrightarrow[4\beta^-]{7\alpha}$	${}^{209}\mathrm{Bi}$	(100%)	$A = 4n + 1$

where n is
an integer

EXTINCT:

TOTAL: 45 NUCLEI ($t_{1/2}$ of all daughters < $t_{1/2}$ of parents)

Radioactive Decay in Thorium and Uranium Series



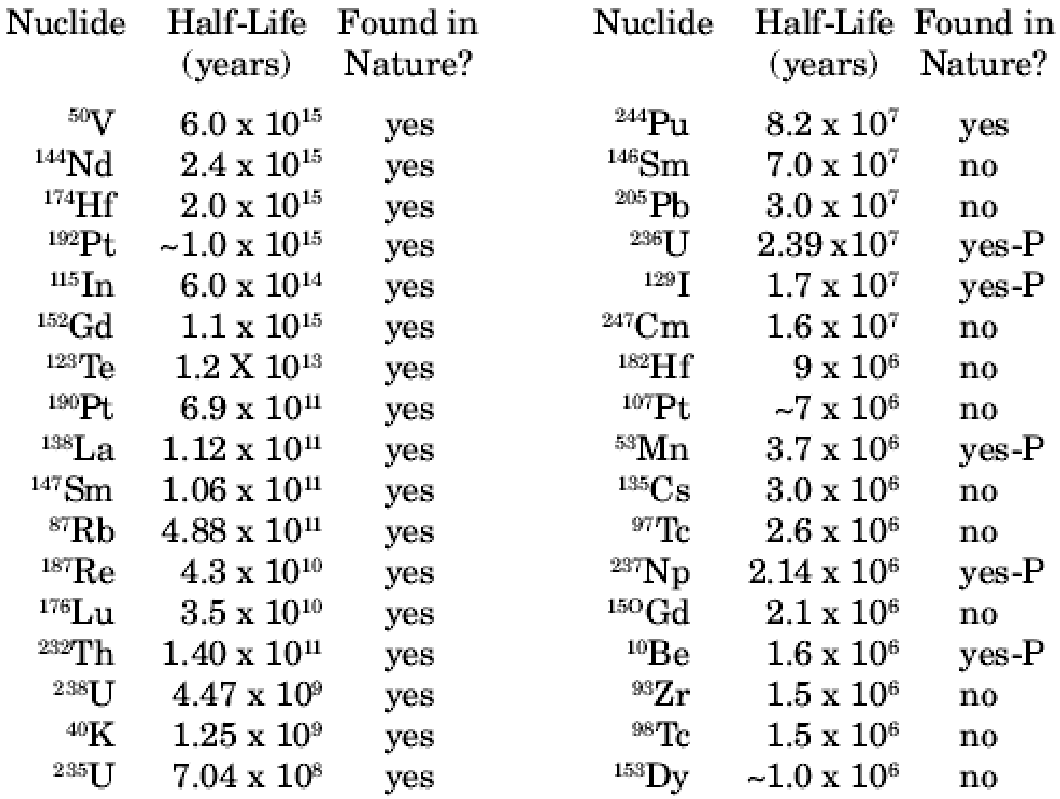
b. Lighter Radionuclides in Nature

(1) Survivors of Nucleosynthesis ; esp. ^{40}K, ^{87}Rb ,^{147}Sm
TOTAL = 15

(2) Cosmic-Ray-Induced Activity
^{3}H(12y), ^{14}C(5280y), ^{7}Be(52d), ^{10}Be(~10^{6}y), …

c. Natural radioactivities carry history of solar system and its evolution

NUCLIDES PRESENT LISTED BY HALF-LIFE*



Nuclide	Half-Life (years)	Found in Nature?	Nuclide	Half-Life (years)	Found in Nature?
^{50}V	6.0×10^{15}	yes	^{244}Pu	8.2×10^{7}	yes
^{144}Nd	2.4×10^{15}	yes	^{146}Sm	7.0×10^{7}	no
^{174}Hf	2.0×10^{15}	yes	^{205}Pb	3.0×10^{7}	no
^{192}Pt	$\sim 1.0 \times 10^{15}$	yes	^{236}U	2.39×10^{7}	yes-P
^{115}In	6.0×10^{14}	yes	^{129}I	1.7×10^{7}	yes-P
^{152}Gd	1.1×10^{15}	yes	^{247}Cm	1.6×10^{7}	no
^{123}Te	$1.2 \mathrm{X} 10^{13}$	yes	^{182}Hf	9×10^{6}	no
^{190}Pt	6.9×10^{11}	yes	^{107}Pt	$\sim 7 \times 10^{6}$	no
^{138}La	1.12×10^{11}	yes	^{53}Mn	3.7×10^{6}	yes-P
^{147}Sm	1.06×10^{11}	yes	^{135}Cs	3.0×10^{6}	no
^{87}Rb	4.88×10^{11}	yes	^{97}Tc	2.6×10^{6}	no
^{187}Re	4.3×10^{10}	yes	^{237}Np	2.14×10^{6}	yes-P
^{176}Lu	3.5×10^{10}	yes	^{150}Gd	2.1×10^{6}	no
^{232}Th	1.40×10^{11}	yes	^{10}Be	1.6×10^{6}	yes-P
^{238}U	4.47×10^{9}	yes	^{93}Zr	1.5×10^{6}	no
^{40}K	1.25×10^{9}	yes	^{98}Tc	1.5×10^{6}	no
^{235}U	7.04×10^{8}	yes	^{153}Dy	$\sim 1.0 \times 10^{6}$	no

"Yes" indicates that an isotope is found in some quantity in nature. "Yes-P" indicates that the isotope is present, but it is produced by the decay of another, longer-lived isotope.

5. Synthetic Nuclei ($t_{1/2} \lesssim 10^8$y)
Isotopes of all elements: Z = 0 to 117 (and more?)

6. Grand Total: ≈ 3500 nuclei and still counting

Factors that Govern Decay Rate

1. Energetics

large Q ⇒ rapid decay (short half-life)

$$A \rightarrow B + C + Q$$

2. Quantum Structure

Spin and Parity: Changes in Iπ between parent and daughter slow down decay rate

e.g.	3 $s_{1/2}$	➔	2 $d_{5/2}$	➔	1 $p_{3/2}$
	Iπ = 1/2+		5/2+		3/2-
	π even		even		odd

II. Alpha Decay

A. Mechanism:

$$ {}^{A}_{Z}X \rightarrow {}^{4}_{2}He + {}^{A-4}_{Z-2}Y + Q_{\alpha} $$

He^{2+} Y^{2-} ← Atomic Ionization State

Alpha Recoil

B. Energetics

1. Spectra: Discrete energies
2. $Q_{\alpha} = \Delta(X) - \Delta(Y) - \Delta(\alpha)$
3. Energy systematics

   Range of values: $Q_{\alpha} \sim 1.5\text{-}12$ MeV measured
4. ${}^{228}Th$ Example

$$ {}^{228}_{90}Th \rightarrow {}^{224}_{88}Ra + {}^{4}_{2}He + Q_{\alpha} $$

$$ Q_{\alpha} = \Delta({}^{228}Th) - \Delta({}^{224}Ra) - \Delta(\alpha) = 26.758 - 18.313 - 2.425 = 5.520 \text{ MeV} $$

Measure: $E_{\alpha} = 5.423$ MeV WHY?

5. Disposition of Q_α

(1) Kinetic energy of α + recoil : $E_\alpha + E_R$

(2) Internal excitation energy ($\equiv$ heat) of recoil nucleus, E^*
(α has no stable excited states)

<u>Case I:</u> $Q_\alpha \Rightarrow$ Kinetic Energy Only

$$\alpha \Leftarrow \quad X \quad \Rightarrow \quad Y$$

X, Y α all in lowest (ground) energy state
i.e., $E^* = 0$ $(T = 0)$

- Energy Conservation: $Q_\alpha = E_\alpha + E_R$ $(E = 1/2\ Mv^2)$
- Linear Momentum Conservation

$$\overrightarrow{p_\alpha} + \overrightarrow{p_R} = 0$$

$$p = \sqrt{2ME}$$

Result of energy and linear momentum conservation

$$\boxed{E_\alpha = \frac{M_R}{M_R + M_\alpha} Q_\alpha \approx \frac{A_R}{A_R + A_\alpha} Q_\alpha}$$

$$\boxed{E_R = \frac{M_\alpha}{M_R + M_\alpha} Q_\alpha \approx \frac{A_\alpha}{A_R + A_\alpha} Q_\alpha}$$

True for all
2-body
breakup
processes

SPECTRA MUST BE DISCRETE, since A, Q, M are all constants

Tag for nucleus ID

b. <u>Case II:</u> Decay to Excited States

$$\alpha \Leftarrow \quad X \quad \Rightarrow \quad Y^*$$

$E^* \Rightarrow E\gamma$ i.e., system then undergoes γ-decay

Kinetic energy of alpha particle : E'_a

Kinetic energy of recoil nucleus: E'_R

- Energy Conservation:

$$Q_\alpha = E'_\alpha + E'_R + E^* \quad = \quad E'_\alpha + E'_R + E_\gamma$$

$Q_\gamma \cong E_\gamma$ since $M_\gamma = 0$

- Momentum Conservation

$$0 = p'_{\alpha} + p'_{R} + p_{\gamma} \qquad p_{\gamma} \approx 0 \text{ since } \; ; \; \therefore \text{ neglect}$$

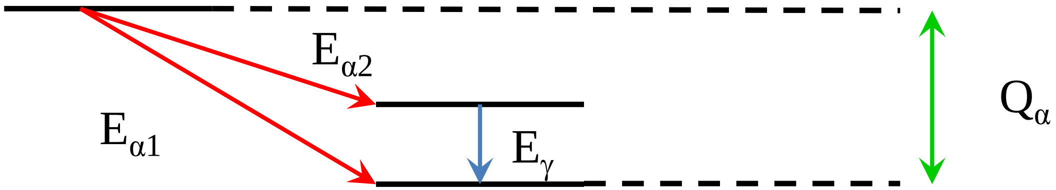
$$0 = p'_{\alpha} + p'_{R}$$

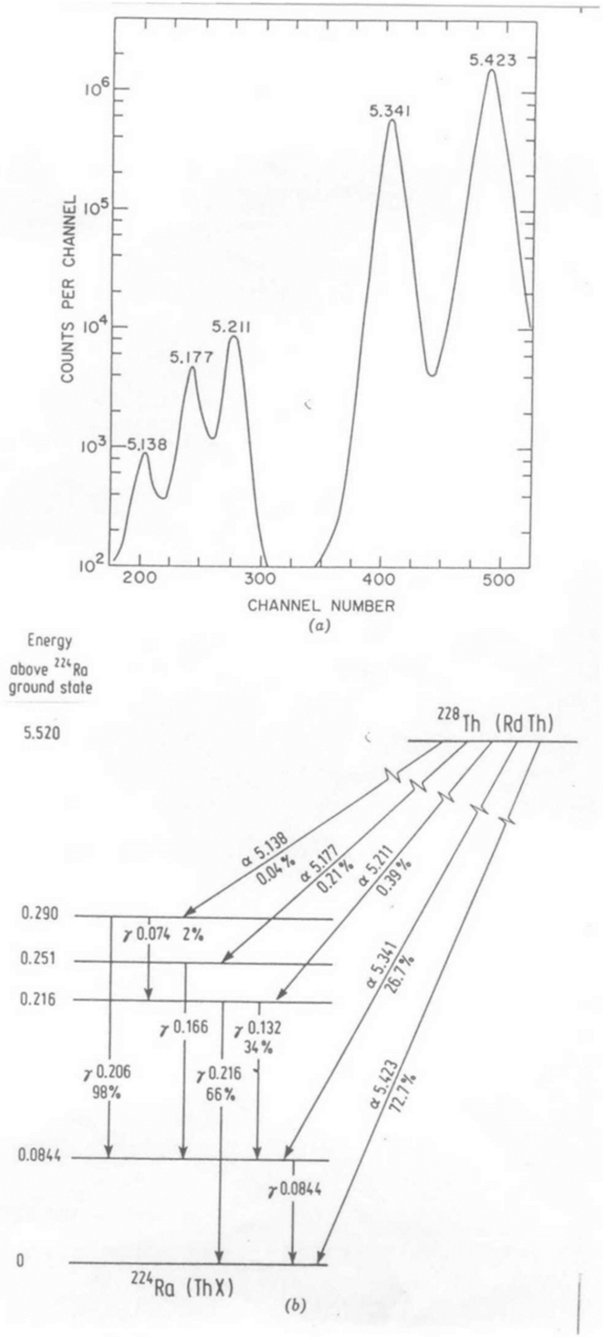
Result of energy and linear momentum conservation for this case:

$$\boxed{E_{\alpha} = \frac{M_R}{M_R + M_{\alpha}}(Q_{\alpha} - E_{\gamma}) \approx \frac{A_R}{A_R + A_{\alpha}}(Q_{\alpha} - E_{\gamma})}$$

$$\boxed{E_{R} = \frac{M_{\alpha}}{M_R + M_{\alpha}}(Q_{\alpha} - E_{\gamma}) \approx \frac{A_{\alpha}}{A_R + A_{\alpha}}(Q_{\alpha} - E_{\gamma})}$$

NOTE: TOTAL ENERGY MUST BE THE SAME, REGARDLESS OF PATHWAY




COUNTS PER CHANNEL
10^2
10^3
10^4
10^5
10^6
5.138
5.177
5.211
5.341
5.423
200
300
400
500
CHANNEL NUMBER
(a)
Energy above ^{224}Ra ground state
5.520
0.290
0.251
0.216
0.0844
0
^{228}Th (Rd Th)
α 5.138 0.04%
α 5.177 0.21%
α 5.211 0.39%
α 5.341 26.7%
α 5.423 72.7%
γ 0.074 2%
γ 0.206 98%
γ 0.166
γ 0.216 66%
γ 0.132 34%
γ 0.0844
^{224}Ra (ThX)
(b)

C. Alpha Decay Probability

1. Energetics: Q_α positive for all A>140 nuclei

2. Range of Measured Half-Lives (~10^{44})

10^{16} y > $t_{1/2}$ > 10^{-21} s

3. Why α ?

a. Proton & Neutron Emission: Q_p , Q_n are negative near valley of beta stability (peak of peninsula); Thermodynamically forbidden

b. Other Nuclei ; e.g. ^{12}C, ^{16}O …

- $Q(^{12}C)$, $Q(^{16}O)$ positive ; therefore possible
- Probability is low (i.e., $t_{1/2}$) is long) -- $P(^{14}C)/P\alpha \sim P^{-10}$

(Exotic decay mode)

$$^{232}_{90}Th \rightarrow {}^{14}_{6}C + {}^{218}_{84}Po$$

FACTS

- 11.7 MeV α particles from ^{212m} Po are the highest energy alphas from a radioactive source
- 2.0 MeV alphas from Sm are the among lowest energy alphas from a radioactive source
- Most alpha particles from radioactive sources fall in the range of 4-8 MeV.
- **Associated with this narrow range in energy is the enormous range in half-life noted above**.

WHY?