Let’s consider what we learned in the last lecture.

\[ ^{238}\text{U} \rightarrow ^{234}\text{Th} + ^{4}\text{He} + Q_{\alpha} \]

Conservation of energy (and equivalence with mass) yields:

\[ Q_{\alpha} = \Delta(^{238}\text{U}) - (\Delta(^{234}\text{Th}) + \Delta(^{4}\text{He})) \]

\[ Q_{\alpha} = (47.305) - (40.610) - (2.425) = 4.27 \text{ MeV} \]

\( Q_{\alpha} > 0 \). What does this tell us?

\[ ^{238}\text{U} \rightarrow ^{237}\text{U} + n \]

\[ Q_{n} = \Delta(^{238}\text{U}) - (\Delta(^{237}\text{U}) + \Delta(n)) \]

\[ Q_{n} = (47.305) - (45.385) - (8.071) = -6.151 \text{ MeV} \]

\( Q_{n} < 0 \). What does this tell us?

\[ ^{238}\text{U} \rightarrow ^{237}\text{Pa} + ^{1}\text{H} \]

\[ Q_{p} = (47.305) - (47.6) - (7.289) = -7.584 \text{ MeV} \]

\( Q_{p} < 0 \). What does this tell us?
Facts from the last lecture:

- \( Q_\alpha > 0 \) for all \( A > 140 \) nuclei; They range from 1.5 to 12 MeV
- Alpha decay half-lives range from \( 10^{16}\text{y} > t_{1/2} > 10^{-21} \text{s} \)
- WHY?

Draw a picture to explain why \( Q_\alpha \) falls in such a small range while the range for \( t_{1/2} \) is so large. Clearly label all important aspects of your picture. Then explain your picture in a short paragraph. In explaining your picture you should reference the concepts of thermodynamic stability and kinetic stability and make use of the discussion in class about Q values.
Beta-Decay: Neutron-Proton Transformations

Connects Isobars

charge

NEUTRON EXCESS

Negatron Decay

\( n \rightarrow ^1H + \beta^- + \bar{\nu} \)

PROTON EXCESS

Positron/electron capture Decay

\( ^1H \rightarrow ^1n + \beta^+ + \nu \)
\( ^1H + e^- \rightarrow ^1n + \nu \)
A. Beta-decay Half-lives

1. Factors that Influence Rate
   a. **WEAK NUCLEAR FORCE:**
      
      \[ \therefore \text{SLOW} \]  \( t_{1/2}(\beta^\pm) \geq 10^{-3} \text{ s} \) (cf \( \approx 10^{-21} \text{ s} \) for \( \alpha \) )
   b. **LARGE \( Q_{\beta^-} \)** \( \Rightarrow \) Short half-life
   c. Structure: spin and parity changes (\( \Delta I \& \Delta \pi \)) \( \Rightarrow \) retard rates

2. **Example**

\[
\begin{align*}
24_{11}\text{Na} & \rightarrow 24_{12}\text{Mg} + \beta^- + \bar{\nu} \quad \bigg\}\begin{array}{l}
4^+ \rightarrow 4^+ \text{ preferred by structure (99.9%)} \\
4^+ \rightarrow 0^+ \text{ preferred by energetics}
\end{array}
\end{align*}
\]
B. Negatron Decay

Negatron \( \equiv {0\beta^-}_{-1} \) (i.e., electron of nuclear origin)

1. Mechanism

\[
\begin{bmatrix}
\text{AX} & \text{A}_Y + {0\beta^-}_{-1} + \bar{\nu} + Q_{\beta^-}
\end{bmatrix}
\]

[\( Y^{+1} \)] is initial chemical state-cation

Nuclear reactors: weird cation chemistry

2. Examples

\[
\begin{align*}
{3\text{H}} & \rightarrow {3\text{He}} + {0\beta^-}_{-1} + \bar{\nu} \\
{14\text{C}} & \rightarrow {14\text{N}} + {0\beta^-}_{-1} + \bar{\nu}
\end{align*}
\]

3. Energetics

a. \( Q_{\beta^-} = \Delta(A\text{X}) - [\Delta(A\text{Y})^+ + \Delta_{e^-} + \Delta_{\nu}] \)

\( \Delta_{\nu} \approx 0 \); 

\( \Delta(A\text{Y}) = \Delta(A\text{Y}^+) + \Delta_{e^-} \)

\( Q_{\beta^-} = \Delta(A\text{X}) - \Delta(A\text{Y}) \)

\( \therefore \) IF \( \Delta(X) > \Delta(Y) \); \( \beta^- \) emission is possible

b. Observed Q-values: \( Q_{\beta^-} \approx 5 \) MeV \((\beta^- \) is relativistic)

c. Spectrum

Spectrum is continuous instead of discrete lines as in \( \alpha \) (and \( \gamma \)) decay. First evidence for neutrinos. If 2-body, then \( E_{\beta^-} \approx Q_{\beta^-} \)

For a three-body breakup, energy and momentum can be shared in an infinite number of ways;
C. Positron Decay: $^0_1\beta^+$ (antiparticle of electron)

1. Mechanism:

\[
\left[ \begin{array}{c} Z \, X \rightarrow 0 \beta^+ + \begin{array}{c} A \, Y \\ Z-1 \end{array} \end{array} \right] \]

\[ [Y^{-1}] \text{ is initial chemical state ; anion} \]

\[ \text{CHARACTERISTIC OF PROTON-RICH NUCLEI; OCCURS INSIDE NUCLEUS} \]

2. Example: \[ ^{11}_6 \text{C} \rightarrow ^{0}_{\text{B}} \beta^+ + ^{11}_5 \text{B} + \nu + Q_{\beta^+} \] (t$_{1/2}$ = 21 min)

Used to tag biomolecules; $^{13}$N + $^{15}$O also, medical cyclotron

3. Energetics

a. \[ Q_{\beta^+} = \Delta(X) - [\Delta(Y^-) + \Delta \beta^+ + \Delta \nu] \; ; \Delta(e^-) = \Delta(\beta^+) \]

\[ Q_{\beta^+} = \Delta(X) - \Delta(Y) - 1.022 \text{ MeV} \]

\[ Q_{\beta^+} = \Delta(X) - \Delta(Y) - 1.022 \text{ MeV} \]
Figure 2.4. Decay scheme for a radionuclide undergoing negatron decay. Two $\beta$ decay paths are shown. A $\gamma$-ray is emitted from an excited state of the daughter in one decay path.
NOTE: Mass of antiparticle in matter world is equal to the mass of the particle. Creation requires twice the mass of the particle to raise it from antimatter state; energy is stolen from the rest of the nucleus.

b. Result: For $\beta^+$ decay to occur:

$$\Delta(X) - \Delta(Y) \geq 1.022 \text{ MeV}$$

c. $Q_{\beta^+} \approx 5 \text{ MeV}$

4. Relative Spectrum Shape
Coulomb field of nucleus attracts $\beta^-$, but repels $\beta^+; \text{ shape is distorted}$

5. Fate of Positron
a. Thermalization: collisions with other electrons reduce kinetic energy to room temperature (3 kT/2)

b. Positronium Formation: (Lightest molecule)

$$e^- + e^+ ; t_{1/2} \sim 10^{-10} \text{ s} \begin{cases} \text{ortho} \uparrow \downarrow ; J = 1 \\ \text{para} \downarrow \uparrow ; J = 0 \end{cases}$$

Lifetime depends on chemical environment

c. Annihilation

$$\{ e^+ + e^- \rightarrow 2\gamma \}; \ E_\gamma = 0.511 \text{ MeV} = M_e c^2$$

Annihilation radiation (180° apart)

d. Radioactive Tag Highly specific

- BACK-TO-BACK $\gamma$s
- Monoenergetic 0.511 MeV gamma
- Coincidence detection
- PET: positron emission tomography
C. Electron Capture Decay -- EC
Alternate mechanism to positron decay for proton-rich nuclei.
SAME NET RESULT for DAUGHTER

1. Mechanism

\[
\begin{align*}
\nu & \text{ violates mass-energy} \\
& \text{unless inside nucleus}
\end{align*}
\]

\[
\begin{align*}
\frac{1}{2}H + \frac{1}{2}e^- & \rightarrow \frac{1}{2}n + \nu + Q_{EC} \\
\frac{1}{2}H & \quad \text{neutral}
\end{align*}
\]

\[
\begin{align*}
\frac{A}{Z}X^+ + \frac{0}{-1}e^- & \rightarrow \frac{A}{Z-1}Y + \nu + Q_{EC} \\
\frac{A}{Z}X & \quad \text{neutral}
\end{align*}
\]

b. Example: \( \frac{37}{18}Ar + \frac{0}{-1}e^- \rightarrow \frac{37}{17}Cl + \nu \); \( t_{1/2} = 35.0 \text{ d} \)

reverse reaction of solar neutrino experiment
c. \[ Q_{EC} = \Delta(x) - \Delta(y) \] \[ \therefore \text{If } \Delta(x) > \Delta(y) \text{ EC is possible} \]

Note: \[ Q_{EC} = Q_{\beta^+} + 1.022 \text{ MeV} \]

2. Atomic Effects
   a. Capture Process: occurs preferentially from atomic orbitals nearest nucleus (low n) – highest probability for being inside the nucleus:

   \[
   \begin{align*}
   &\{1s, 2s, 3s, \ldots\} \\
   &\text{K-shell, L-shell, M-shell} \\
   \end{align*}
   \]

   RESULT: Capture Probability \( P_{EC(K)} > P_{EC(L)} > P_{EC(M)} \) … etc.

   b. For fixed principal quantum number \( n \), capture occurs from lowest \( \ell \) state; i.e., \( s \) state has a higher probability of being inside nucleus.

   \[ \Rightarrow P_{EC(ns)} > P_{EC(np)} > P_{EC(nd)} \text{ etc} \]

   \[ s \quad p \quad d \]

   c. \( ^{37}\text{Ar} \) Example

   \[ ^{37}_{18}\text{Ar} \ (1s^22s^22p^63s^23p^6) \Rightarrow ^{37}\text{Cl} [1s^12s^22p^63s^23p^6] \]

3. Radiation from EC
   a. Primary: monoenergetic neutrino (hard to detect)
      heavy recoil nucleus (hard to detect)
b. **Secondary: Detectable Radiation**

- **x-rays** – emitted during electronic orbital rearrangement
  x-rays are characteristic of DAUGHTER nucleus. **WHY?**
- **Auger electrons** – x-rays may interact with outer orbital electrons and eject low energy electrons instead (internal photoelectric effect)

\[
\frac{P(x\text{-ray})}{P(\text{Auger})} = aZ
\]

REASON: x-ray energy increases with Z; \(\therefore\) \(\lambda\) too short to interact.

- **Gamma rays** – If EC populates an excited state of daughter nucleus, monoenergetic \(\gamma\)-ray will be emitted.

c. **EC most difficult to detect of all decay modes**

4. **Chemical Effects**

EC is only decay mode affected by the chemical environment

a. **Ligand dependence:** \(t_{1/2}(\text{Be}) < t_{1/2}(\text{BeF}_2)\): \(\text{F} \leftarrow \text{Be} \rightarrow \text{F}\)

b. **Pressure dependence:** High pressure shortens half-life

c. **Stars:** \(4^7\text{Be}\) cannot decay, no \(e^-\)s; \(Q_{\beta^+} = -0.16\) MeV

5. **EC/\(\beta^+\) Competition**

a. If \(Q_{\text{EC}} < 1.022\) MeV, EC ONLY
If \(Q_{\text{EC}} \geq 1.022\) MeV, EC & \(\beta^+\)

b. \[
\frac{P(\text{EC})}{P(\beta^+)} = aZ \quad \text{if} \quad Q_{\text{EC}} \geq 1.022\ \text{MeV} \; \text{;} \quad \text{WHY?}
\]

For \(A \gtrsim 180\), EC predominates; factors

- Coulomb barrier inhibits \(\beta^+\)
- Probability that 1s electron is inside the nucleus increases with increasing \(Z\) and \(A\).